Worksheet 7 – Polynomial Functions (§4.2 and §4.4)

1. Compute the zeros of the given polynomial and their corresponding multiplicities. State the leading term of the polynomial. Then use this information to sketch the graph of the polynomial.

(a) $p(x) = x(x+2)^2$
(b) $p(x) = x(x+2)^3$
(c) $p(x) = -2(x-2)^2(x+1)$
(d) $p(x) = (2x+1)^2(x-3)$
(e) $p(x) = x^3(x+2)^2$
(f) $p(x) = (x-1)(x-2)(x-3)(x-4)$
(g) $p(x) = (x+5)^2(x-3)^4$
(h) $p(x) = x^2(x-2)^2(x+2)^2$

2. Use long division to divide the given polynomials. Write your result as "dividend = divisor x quotient + remainder."

(a) $\frac{4x^2+3x-1}{x-3}$
(b) $\frac{2x^3-x+1}{x^2+x+1}$
(c) $\frac{5x^4-3x^3+2x^2-1}{x^2+4}$
(d) $\frac{-x^5+7x^3-x}{x^3-x^2+1}$

3. Use synthetic division to divide the given polynomials. Write your result as "dividend = divisor x quotient + remainder."

(a) $\frac{3x^2-2x+1}{x-1}$
(b) $\frac{x^2-5}{x-5}$
(c) $\frac{x^3+8}{x+2}$
(d) $\frac{4x^3+2x-3}{x-3}$
(e) $\frac{18x^2-15x-25}{x-\frac{5}{3}}$
(f) $\frac{4x^2-1}{x-\frac{1}{2}}$
(g) $\frac{x^4-6x^2+9}{x-\sqrt{3}}$
(h) $\frac{x^6-6x^4+12x^2-8}{x+\sqrt{2}}$

4. For each given polynomial $p(x)$ and given number c, use the Remainder Theorem to compute $p(c)$. If $p(c) = 0$, then factor $p(x)$.

(a) $p(x) = 2x^2 - x + 1$, $c = 4$
(b) $p(x) = 4x^2 - 33x - 180$, $c = 12$
(c) $p(x) = 2x^3 - x + 6$, $c = -3$
(d) $p(x) = x^4 - 2x^2 + 4$, $c = \frac{3}{2}$
(e) $p(x) = x^3 + 2x^2 + 3x + 4$, $c = -1$
(f) $p(x) = 3x^3 - 6x^2 + 4x - 8$, $c = 2$
(g) $p(x) = 8x^3 + 12x^2 + 6x + 1$, $c = -\frac{1}{2}$
(h) $p(x) = x^2 - 4x + 1$, $c = 2 - \sqrt{3}$

5. Factor each given polynomial completely.

(a) $p(x) = x^3 - 7x + 6$
(b) $p(x) = x^4 + x^3 - x^2 + x - 2$
(c) $p(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$
(d) $p(x) = x^5 + x^4 - 10x^3 + 8x^2$