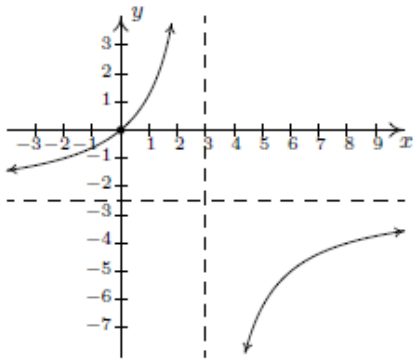
Worksheet 8 KEY – Rational Functions (§4.5)

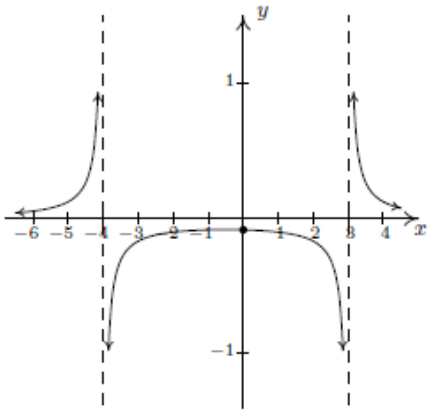
1.

$f(x) = \dfrac{5x}{6-2x}$
Domain: $(-\infty, 3) \cup (3, \infty)$
No holes in the graph
x-intercept: $(0, 0)$
y-intercept: $(0, 0)$
Vertical asymptote: $x = 3$
Horizontal asymptote: $y = -\frac{5}{2}$



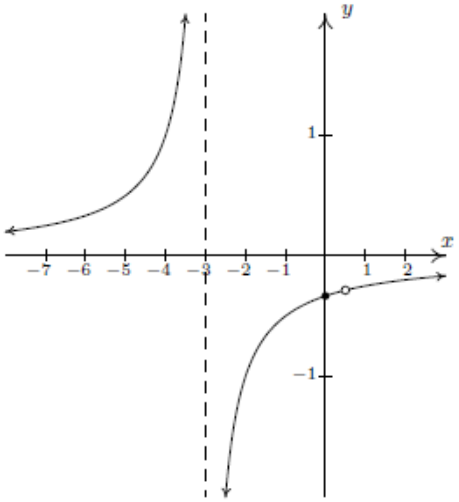
2.

$f(x) = \dfrac{1}{x^2+x-12} = \dfrac{1}{(x-3)(x+4)}$
Domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$
No holes in the graph
No x-intercepts
y-intercept: $(0, -\frac{1}{12})$
Vertical asymptotes: $x = -4$ and $x = 3$
Horizontal asymptote: $y = 0$



3.

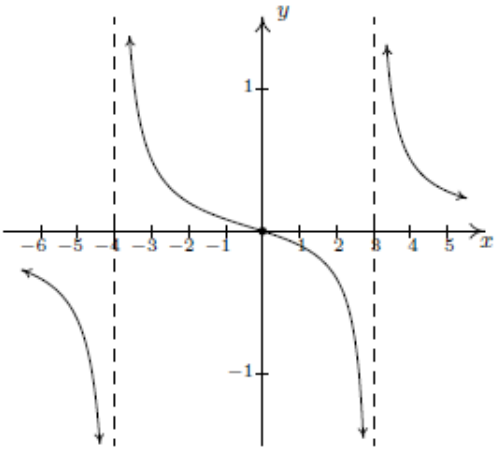
$f(x) = \dfrac{2x-1}{-2x^2-5x+3} = -\dfrac{2x-1}{(2x-1)(x+3)}$
Domain: $(-\infty, -3) \cup (-3, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$
$f(x) = \dfrac{-1}{x+3}$, $x \neq \frac{1}{2}$
Hole in the graph at $(\frac{1}{2}, -\frac{2}{7})$
No x-intercepts
y-intercept: $(0, -\frac{1}{3})$
Vertical asymptote: $x = -3$
Horizontal asymptote: $y = 0$



4.

$$f(x) = \frac{x}{x^2 + x - 12} = \frac{x}{(x-3)(x+4)}$$

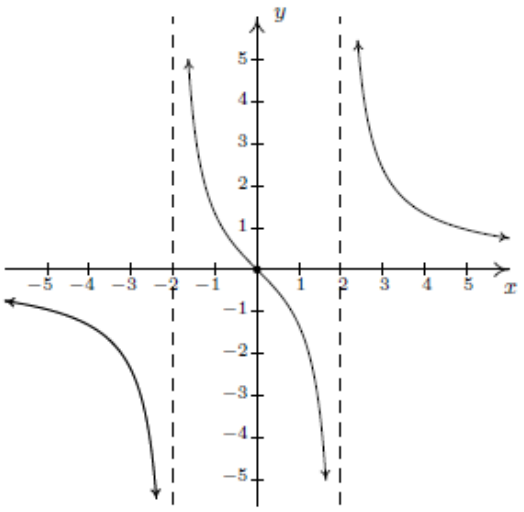
Domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$
No holes in the graph
x-intercept: $(0, 0)$
y-intercept: $(0, 0)$
Vertical asymptotes: $x = -4$ and $x = 3$
Horizontal asymptote: $y = 0$



5.

$$f(x) = \frac{4x}{x^2 - 4} = \frac{4x}{(x+2)(x-2)}$$

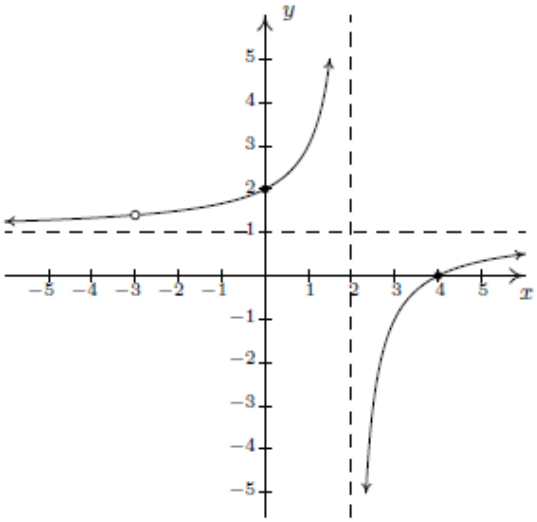
Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
No holes in the graph
x-intercept: $(0, 0)$
y-intercept: $(0, 0)$
Vertical asymptotes: $x = -2, x = 2$
Horizontal asymptote: $y = 0$



6.

$$f(x) = \frac{x^2 - x - 12}{x^2 + x - 6} = \frac{x-4}{x-2} \; x \neq -3$$

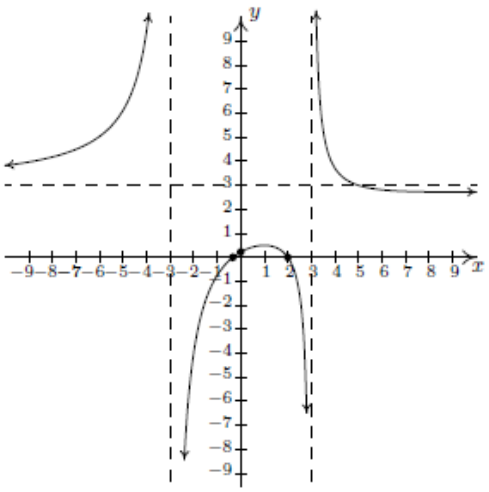
Domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$
Hole at $\left(-3, \frac{7}{5}\right)$
x-intercept: $(4, 0)$
y-intercept: $(0, 2)$
Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 1$



7.

$$f(x) = \frac{3x^2 - 5x - 2}{x^2 - 9} = \frac{(3x+1)(x-2)}{(x+3)(x-3)}$$

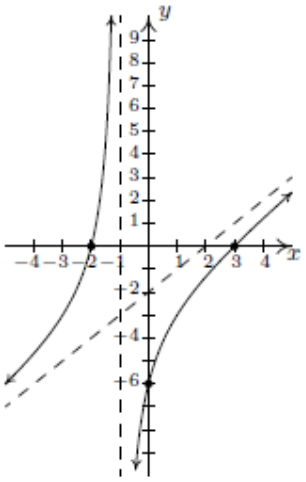
Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
No holes in the graph
x-intercepts: $\left(-\frac{1}{3}, 0\right)$, $(2, 0)$
y-intercept: $\left(0, \frac{2}{9}\right)$
Vertical asymptotes: $x = -3, x = 3$
Horizontal asymptote: $y = 3$



8.

$$f(x) = \frac{x^2 - x - 6}{x + 1} = \frac{(x-3)(x+2)}{x+1}$$

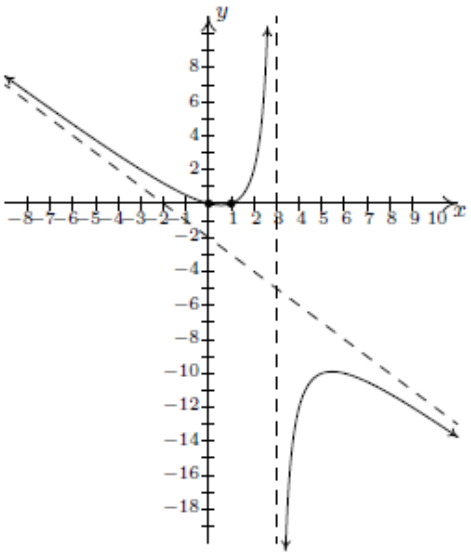
Domain: $(-\infty, -1) \cup (-1, \infty)$
No holes in the graph
x-intercepts: $(-2, 0)$, $(3, 0)$
y-intercept: $(0, -6)$
Vertical asymptote: $x = -1$
Slant asymptote: $y = x - 2$



9.

$$f(x) = \frac{x^2 - x}{3 - x} = \frac{x(x-1)}{3-x}$$

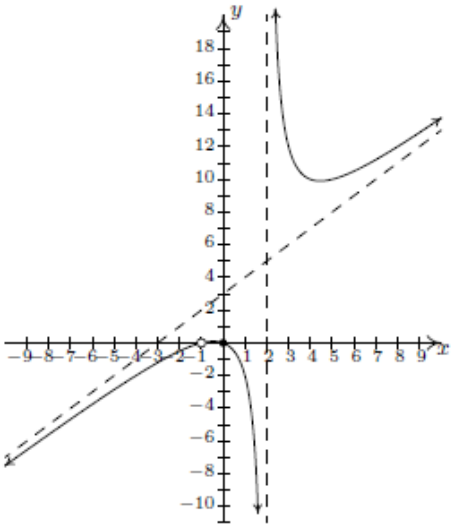
Domain: $(-\infty, 3) \cup (3, \infty)$
No holes in the graph
x-intercepts: $(0, 0)$, $(1, 0)$
y-intercept: $(0, 0)$
Vertical asymptote: $x = 3$
Slant asymptote: $y = -x - 2$



10.

$$f(x) = \frac{x^3 + 2x^2 + x}{x^2 - x - 2} = \frac{x(x+1)}{x-2} \quad x \neq -1$$

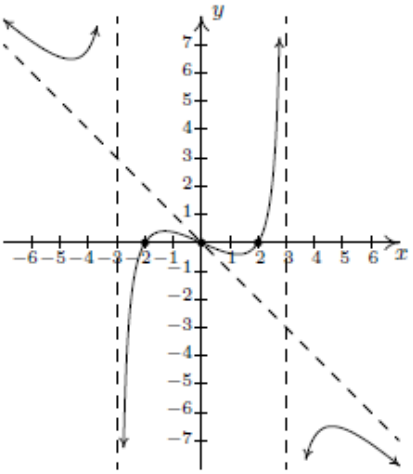
Domain: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$
Hole at $(-1, 0)$
x-intercept: $(0, 0)$
y-intercept: $(0, 0)$
Vertical asymptote: $x = 2$
Slant asymptote: $y = x + 3$



11.

$$f(x) = \frac{-x^3 + 4x}{x^2 - 9}$$

Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
No holes in the graph
x-intercepts: $(-2, 0), (0, 0), (2, 0)$
y-intercept: $(0, 0)$
Vertical asymptotes: $x = -3, x = 3$
Slant asymptote: $y = -x$



12.

$$f(x) = \frac{x^2 - 2x + 1}{x^3 + x^2 - 2x}$$

Domain: $(-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, \infty)$

$$f(x) = \frac{x - 1}{x(x+2)}, \quad x \neq 1$$

Hole in the graph at $(1, 0)$
No x-intercepts
No y-intercepts
Vertical asymptotes: $x = -2$ and $x = 0$
Horizontal asymptote: $y = 0$