Calculus I — Russell Brown
Exam 1 — 18 September 2001

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities.
When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*.
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.
There are 105 points on this test, however no student will be given a grade of more than 100.

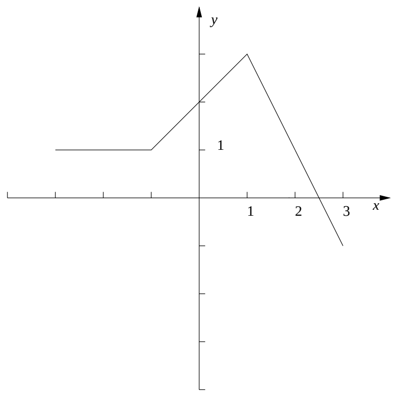
Name ______________________

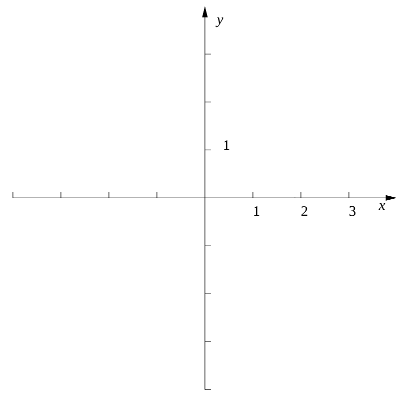
Section ____________

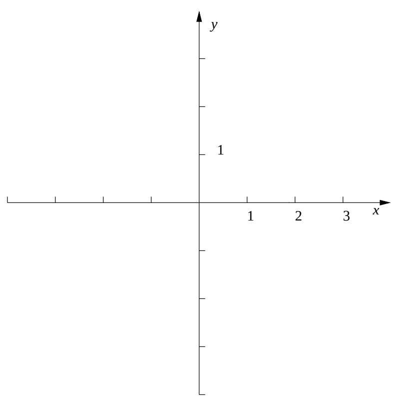
Question	Score	Total
1		10
2		10
3		5
4		10
5		15
6		5
7		10
8		10
9		15
10		10
11		5
Total		100

1. (a) Below is the graph of $f(x)$. On the axes provided, sketch the graphs of $g(x) = f(x+2)$ and $h(x) = f(x-2)+1$. Indicate clearly which is the graph of g and which is the graph of h.

(b) Give the values of $g(1)$ and $h(1)$.







2. (a) Give the formal definition of an odd function. Your answer should be a complete sentence.

   (b) Is $\sin(x)$ odd or even?

   (c) Is $\cos(x)$ odd or even?

   (d) Suppose that f is an odd function. Determine which of the following functions are odd and which are even. Justify your answers.

      i. $f(\sin x)$

      ii. $\cos(f(x))$

      iii. $f(x) \cdot \sin(x)$

3. (a) If $\cos(\theta) = 1/5$ and $3\pi/2 < \theta < 2\pi$, find $\sin(\theta)$.
   (b) With θ as in part a) find $\tan(\theta)$ and $\sec(\theta)$.

4. For each of the following limits, determine its value, if possible. Some of the limits may be infinite. If a limit does not exist, say so. You do not need to justify each step.

   (a) $\lim_{x \to 2} \dfrac{x-2}{x^2-4}$

   (b) $\lim_{x \to 2} \dfrac{x^2+4}{x+2}$.

   (c) $\lim_{x \to 2^+} \dfrac{x}{x-2}$

   (d) $\lim_{x \to 2^-} \dfrac{x}{x-2}$

   (e) $\lim_{x \to 2} \dfrac{x}{x-2}$

5. (a) State the rigorous or ϵ, δ definition of what it means for

$$\lim_{x \to a} f(x) = L.$$

(b) If $|x - 4| < 1$, find a number C so that $|x + 2| < C$.

(c) Use the ϵ, δ definition of limit to show that

$$\lim_{x \to 4} x^2 - 2x = 8.$$

6. Suppose that c is a constant and let

$$f(x) = \begin{cases} 2x+3, & x > 3 \\ cx^2, & x < 3 \end{cases}$$

(a) Determine the right and left-hand limits,

$$\lim_{x \to 3+} f(x) \qquad \text{and} \qquad \lim_{x \to 3^-} f(x).$$

(Hint: One of your answers will involve c.)

(b) Find c so that

$$\lim_{x \to 3} f(x)$$

exists.

7. Below is the graph of a function f. Answer the following.

(a) Determine if the following limits exist and give the value, if the limit exists. If a limit does not exist, write down that the limit does not exist.

i. $\lim_{x\to -1^+} f(x) =$ ____________

ii. $\lim_{x\to -1^-} f(x) =$ ____________

iii. $\lim_{x\to -1} f(x) =$ ____________

iv. $\lim_{x\to 2^+} f(x) =$ ____________

v. $\lim_{x\to 2^-} f(x) =$ ____________

vi. $\lim_{x\to 2} f(x) =$ ____________

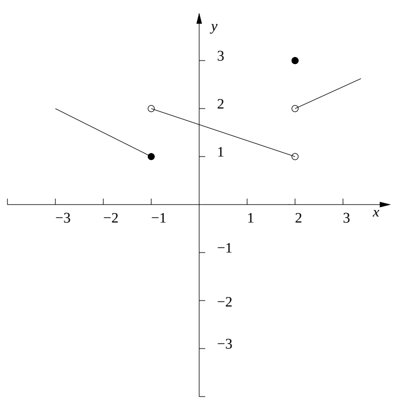
(b) Answer the following questions.

i. Is f continuous at $x = -1$?

ii. Is f continuous from the left at $x = -1$?

iii. Is f continuous from the right at $x = -1$?

iv. Is f continuous at $x = 0$?



8. (a) State the intermediate value theorem.

   (b) Use the intermediate value theorem to show that the equation $x^5 = \sqrt{x^2+1}$ has a root.

9. (a) State the definition of $f'(a)$ the derivative f at $x = a$.

(b) Use the definition to find the derivative of

$$f(x) = \sqrt{x+2}.$$

(c) Use your answer to part b) to find the slope of the tangent line to the graph of $f(x)$ at $x = 2$ and find the equation of the tangent line at $x = 2$.

10. Let $f(x) = x^2$. Use the definition of the derivative to find $f'(x)$.

11. Consider the two graphs below. One is the graph of a function $f(x)$ and the other is the graph of $f'(x)$. Determine which is the graph of f and which is the the graph of f'. Write a sentence to explain your answer.