MA 113 - Calculus I | Fall 2007
EXAM 1 | September 18, 2007

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. check answers when possible,
2. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
3. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question. If you use the back of a sheet, please indicate this by the question.

You are to answer two of the last three questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name ____________________

Section __________

Last four digits of student identification number __________

Question	Score	Total
1		10
2		9
3		7
4		9
5		9
6		9
7		7
8		9
9		14
10		14
11		14
Free	3	3
		100

1. Consider the rational function

$$f(x) = \frac{x-2}{x^2-5x+6}.$$

(a) Determine each of the following limits if it exists.

(i) $\lim_{x\to 1} f(x)$

(ii) $\lim_{x\to 2^-} f(x)$

(iii) $\lim_{x\to 3^+} f(x)$

(b) At which of the numbers 1, 2, and 3 has f a vertical asymptote?

(i) $\lim_{x\to 1} f(x) =$ ________ (ii) $\lim_{x\to 2^-} f(x) =$ ________ $\lim_{x\to 3^+} f(x) =$ ________

(b) f has a vertical asymptote at __________

2. Consider the function $f(x) = (\sqrt{x+2} - 3)^{100}$.

(a) Determine the domain of f.

(b) Find all numbers a so that the function f is continuous at a.

(c) Determine $\lim_{x \to 2} f(x)$.

(a) The domain of f is ____________ (b) f is continuous at ____________

(c) $\lim_{x \to 2} f(x) =$ ____________________

3. Let f and g be two functions such that the following limits exist:

$$\lim_{x\to 2} g(x) = 6 \quad \text{and} \quad \lim_{x\to 2}[xf(x) - 3g(x)] = 2.$$

Show that the limit

$$\lim_{x\to 2} f(x)$$

exists and calculate its value.

$\lim_{x\to 2} f(x) =$ ______________________

4. Let f be a function such that, for all numbers x,

$$2x^3 + 3 \leq f(x) \leq x^6 + 4.$$

Show that $\lim_{x \to 1} f(x)$ exists and find its value.

$\lim_{x \to 1} f(x) =$ ______________________

5. Let c be a number and consider the function

$$f(x) = \begin{cases} cx^2 - 3 & \text{for } x < 2 \\ x - c & \text{for } x > 2 \end{cases}$$

(a) Find all numbers c such that the limit $\lim_{x \to 2} f(x)$ exists.

(b) Is there a number c such that f is continuous at $x = 2$? As always, justify your answer.

(a) $c =$ ____________ (b) yes / no (*circle the correct answer*)

6. Using the definition, find the equation of the tangent line to the curve $y = \sqrt{x+1}$ at $x = 3$. Write your result in the form $y = mx + c$.

The equation of the tangent is ____________________

7. An apple drops from the top of a tree which is 20 meter tall. Assuming that the apple falls $s = 4.9t^2$ meters after t seconds, find:

(a) the distance the apple traveled after 1 second,

(b) the average velocity between 1 and 2 seconds.

(a) The distance is ____________ (b) The average velocity is ____________

8. Consider the function $f(x) = (x+1)^2$.

(a) Determine the slope m_a of the secant line through the points $(1, f(1))$ and $(a, f(a))$ with $a \neq 1$ (as a function of a). As always, simplify your answer.

(b) Find $\lim_{a \to 1} m_a$ if it exists.

(a) $m_a =$ ____________________ (b) $\lim_{a \to 1} m_a =$ ______________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

9. (a) State the principle of mathematical induction.

(b) Prove by mathematical induction that, for all integers $n \geq 1$, the following equality is true:

$$\sum_{k=1}^{n} (2k) = n^2 + n.$$

10. (a) State the Intermediate Value Theorem.

(b) Explain why and how you can use this theorem to show that the equation

$$x^5 - 3x^4 + 1 = 0$$

has a root between 0 and 1.

11. (a) State the definition of the derivative of a function f at a point a.

(b) Consider the function $f(x) = |x - 1|$. Using the definition, decide whether f is differentiable at

(i) $x = 2$,

(ii) $x = 1$.