MA 113 Calculus I — Fall 2015

Exam 1 — Tuesday, 22 September 2015

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-phones during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

[C,C,D,B,B, D,B,C,A,B]

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

1. If $f(x) = x^2 - 1$, find $f(f(0))$.

   (A) -2

   (B) -1

   (C) 0

   (D) 1

   (E) 2

2. Suppose that $f(x) = |x + 1|$ and the domain of f is $(-\infty, -1]$. Find $f^{-1}(2)$.

   (A) -5

   (B) -4

   (C) -3

   (D) -2

   (E) -1

Record the correct answer to the following problems on the front page of this exam.

3. Which function below is equal to 5^x?

   (A) e^{5x}

   (B) $(\ln(x))^5$

   (C) $e^{5\ln(x)}$

   (D) $e^{x\ln(5)}$

   (E) $5^{\ln(x)}$

4. A bug crawls in a counter clockwise direction along a circle centered at the origin and of radius 3 units. The bug begins at the point $(3, 0)$ and crawls for 9 minutes at a rate of 4 units per minute. Give the bug's location after 9 minutes.

   (A) $(\cos(4/3), \sin(4/3))$

   (B) $(3\cos(12), 3\sin(12))$

   (C) $(3\cos(36), 3\sin(36))$

   (D) $(3\cos(4/3), 3\sin(4/3))$

   (E) $(\cos(12), \sin(12))$

Record the correct answer to the following problems on the front page of this exam.

5. Solve $10^{2x+1} = 100$.

   (A) 0

   (B) 1/2

   (C) 1

   (D) 3/2

   (E) 2

6. Find the value of the limit

$$\lim_{x \to 3} \frac{x-3}{x^2 - 2x - 3}.$$

   (A) $+\infty$

   (B) $-\infty$

   (C) The limit does not exist and is not $+\infty$ or $-\infty$.

   (D) 1/4

   (E) 0/0

Record the correct answer to the following problems on the front page of this exam.

7. Suppose that

$$\lim_{x \to 3}(4f(x) - 3) = 1.$$

Find the value of

$$\lim_{x \to 3} f(x).$$

(A) 0

(B) 1

(C) 2

(D) 3

(E) None of the above.

8. Suppose

$$f(x) = \begin{cases} \dfrac{\cos(2x) - 1}{x}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

If f is continuous for all real numbers, what is the value of c?

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

Record the correct answer to the following problems on the front page of this exam.

9. Suppose that f is a continuous function on the interval $[0,5]$ and we know that

$$f(0) = 1, f(1) = -1, f(2) = 1, f(3) = -1, f(4) = 1, \text{ and } f(5) = -1.$$

Which of the following statements will be true for any such f?

(A) There are at least five solutions of the equation $f(x) = 0$ in the interval $[0,5]$.

(B) There are at most five solutions of the equation $f(x) = 0$ in the interval $[0,5]$.

(C) There are exactly five solutions of the equation $f(x) = 0$ in the interval $[0,5]$.

(D) There are no solutions of the equation $f(x) = 0$ in the interval $[0,5]$.

(E) The equation $f(x) = 1$ has exactly three solutions in the interval $[0,5]$.

10. Suppose that the position of a particle at time t seconds is $p(t) = t^3 - 4t^2$ meters to the right of the origin. Find the average velocity of the particle on the interval $[1,3]$.

(A) -6 meters/second

(B) -3 meters/second

(C) 6 meters/second

(D) 3 meters/second

(E) -18 meters/second

Free Response Questions: Show your work!

11. (a) Find the inverse function of the function given by

$$f(x) = \frac{4+3x}{3+2x}.$$

(b) Give the domain and range of the function f^{-1} you find in part (a).

a) $y = \frac{4+3x}{3+2x}$	1 point, method
Solve to obtain $x = \frac{4-3y}{2y-3}$	3 points
Thus the inverse is $f^{-1}(x) = \frac{4-3x}{2x-3}$	2 points, give inverse
b) The domain of f^{-1} is $(-\infty, 3/2) \cup (3/2, \infty)$ or $\{x : x \neq 3/2\}$.	2 points
c) The range of f^{-1} is the domain of f which is $(-\infty, -3/2) \cup (-3/2, \infty)$ or $\{x : x \neq -3/2\}$	2 points

Free Response Questions: Show your work!

12. Find the limits or state that the limit does not exist. In each case, justify your answer.

(Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\displaystyle\lim_{t\to 0} \frac{t}{|t|}$

(b) $\displaystyle\lim_{x\to 0} \frac{x^2 - 2x}{xe^x}$.

a) The limit does not exist. We have $\lim_{t\to 0^\pm} \frac{t}{\lvert t\rvert} = \pm 1$. Since the one-sided limits are different, the limit does not exist.	Answer 3 points 2 points for justification. Also accept a graph showing that the one-sided limits are different.
b) We begin by simplifying $\frac{x^2-2x}{xe^x} = \frac{x-2}{e^x}$.	2 points
Since the function $(x-2)/e^x$ is continuous at 0, we may evaluate the limit by subtitution,	Justification 2 points
$\lim_{x\to 0} \frac{x-2}{e^x} = -2$.	Answer 1 point

Free Response Questions: Show your work!

13. Using the graph of the function f below, complete the following. If the requested information does not exist, write DNE.

    (a) $\lim_{x\to -2^+} f(x) =$ ____________ 1, 1 point, no justification needed

    (b) $\lim_{x\to -2^-} f(x) =$ ____________ -1, 1 point

    (c) $\lim_{x\to -2} f(x) =$ ____________ DNE, 1 point

    (d) $f(-2) =$ ____________ -1, 1 point

    (e) Is f left-continuous at -2? ____________ Yes, 1 point

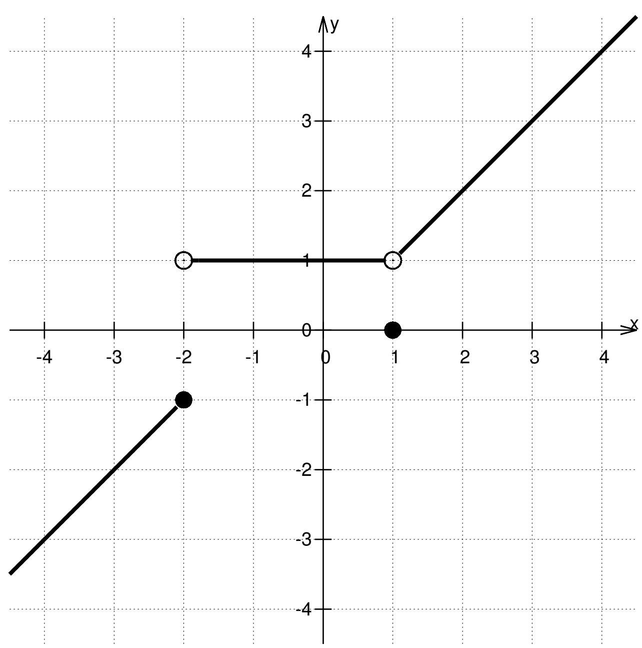
    (f) $\lim_{x\to 1^+} f(x) =$ ____________ 1, 1 point

    (g) $\lim_{x\to 1^-} f(x) =$ ____________ 1, 1 point

    (h) $\lim_{x\to 1} f(x) =$ ____________ 1, 1 point

    (i) $f(1) =$ ____________ 0, 1 point

    (j) Is f left-continuous at 1? ____________ No, 1 point



Free Response Questions: Show your work!

14. (a) State the squeeze theorem.

(b) Use the squeeze theorem to find

$$\lim_{x\to 0} x^2 \cos(1/x).$$

a) Suppose that f, g, and h, are functions defined near a and we have $f(x) \leq g(x) \leq h(x)$ for $x \neq a$ but x near a.	2 points
If we have $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$,	2 points
then $\lim_{x\to a} g(x) = L$	conclusion 1 point Statement should be in complete sentences.
b) Let $f(x) = -x^2$ and $h(x) = x^2$	Correct choice of "squeezing" functions, 2 points.
Since $-x^2 \leq x^2\cos(1/x) \leq x^2$ for $x \neq 0$ and $\lim_{x\to 0} \pm x^2 = 0$	Observing inequalities, 1 point, limits 1 point
we have $\lim_{x\to 0} x^2\cos(1/x) = 0$	Answer 1 point.

Free Response Questions: Show your work!

15. Consider the function $f(x) = 1/(x+3)$.

    (a) Write an expression for the slope of the secant line that passes through the point $(x, f(x))$ and $(-1, f(-1))$.

    (b) Take the limit as x approaches -1 of the expression you found in part (a) to find the slope of the tangent line to the graph of f at $x = -1$.

    (c) Write the equation of the tangent line to the graph of f at $x = -1$ in point-slope form.

a) The slope of the secant line is $\frac{f(x)-f(-1)}{x+1} = \frac{1}{x+1}(\frac{1}{x+3} - \frac{1}{2})$.	2 points, they will need to simplify to evaluate the limit, but we do not require it here.
b) We cannot evaluate the limit of $\frac{1}{x+1}(\frac{1}{x+3} - \frac{1}{2})$ by substitution since the function is not continuous at $x = -1$.	
We begin by simplifying, $\frac{1}{x+1}(\frac{1}{x+3} - \frac{1}{2}) = \frac{-1}{2(x+3)}$.	3 points for simplifying
Since $-1/(2(x+3))$ gives a continuous expression at $x = -1$, we may evaluate the limit by substitution (or the limit laws) to obtain $\lim_{x\to-1} \frac{-1}{2(x+3)} = -1/4$	2 points answer, 1 point justification
c) The tangent line passes through $(-1, 1/2)$ and its slope is $-1/4$. The equation in point-slope form is $y - 1/2 = (-1/4)(x+1)$.	Equation of line, 2 points. Do not deduct if they simplify after writing point slope form.