

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

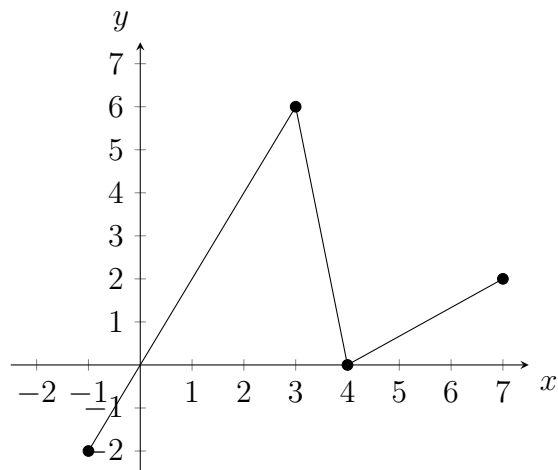
| Question | | | | | |
|----------|---|---|---|---|---|
| 1 | A | B | C | D | E |
| 2 | A | B | C | D | E |
| 3 | A | B | C | D | E |
| 4 | A | B | C | D | E |
| 5 | A | B | C | D | E |
| 6 | A | B | C | D | E |
| 7 | A | B | C | D | E |
| 8 | A | B | C | D | E |
| 9 | A | B | C | D | E |
| 10 | A | B | C | D | E |
| 11 | A | B | C | D | E |
| 12 | A | B | C | D | E |

Exam Scores

| Question | Score | Total |
|----------|-------|-------|
| MC | | 60 |
| 13 | | 10 |
| 14 | | 10 |
| 15 | | 10 |
| 16 | | 10 |
| Total | | 100 |

1. What are the domain and range of $f(x)$ found in the graph below?

- (A) Domain: $[-2, 6]$ Range: $[-1, 7]$
- (B) Domain: $[-1, 7]$ Range: $[-2, 6]$
- (C) Domain: $[0, 7]$ Range: $[0, 6]$
- (D) Domain: $-1, 3, 4, 7$ Range: $-2, 6, 0, 2$
- (E) None of the above



2. The population of a city at time t is $P(t) = 2.8e^{0.004t}$. When will the population double from its initial population at $t = 0$?

- (A) $0.004 \ln(2)$
- (B) $\frac{\ln(2.8)}{0.004}$
- (C) $250 \ln(2.8)$
- (D) $250 \ln(2)$
- (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

3. A bug is located at the point $(5, 0)$ at time $t = 0$ and crawls at the rate of 11 units/minute in the counterclockwise direction along the circle centered at the origin of radius 5. Find the coordinates (x, y) which give the location of the bug after t minutes.

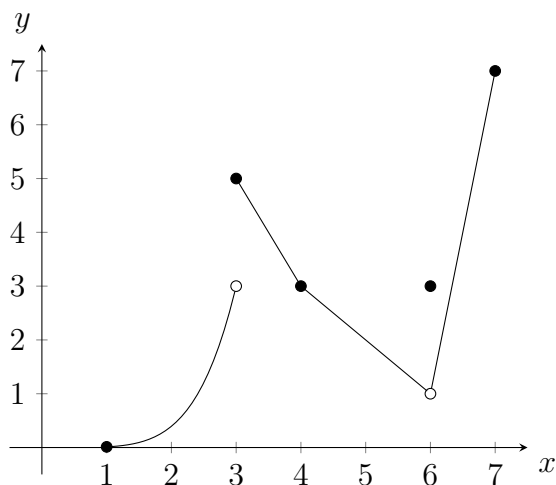
- (A) $(5 \cos(11t/5), 5 \sin(11t/5))$
- (B) $(5 \sin(11t/5), 5 \cos(11t/5))$
- (C) $(11 \cos(5t/11), 11 \sin(5t/11))$
- (D) $(5 \cos(11t), 5 \sin(11t))$
- (E) None of the above

4. A stone is tossed in the air from ground level with an initial velocity of 30 m/s . Its height at time t is $h(t) = 30t - 4.9t^2$. Compute the average velocity of the stone over the time interval $[1.5, 3.5]$.

- (A) 11
- (B) $11/3$
- (C) $1/2$
- (D) $-11/2$
- (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

5. Let $f(x)$ be the function whose graph is shown below. Which of the following limits are all correct?



- (A) $\lim_{x \rightarrow 3^-} f(x) = 3$ and $\lim_{x \rightarrow 4} f(x) = 3$ and $\lim_{x \rightarrow 6^+} f(x) = 1$
 (B) $\lim_{x \rightarrow 3^-} f(x)$ does not exist and $\lim_{x \rightarrow 4} f(x) = 3$ and $\lim_{x \rightarrow 6^+} f(x) = 1$
 (C) $\lim_{x \rightarrow 3^-} f(x) = 5$ and $\lim_{x \rightarrow 4} f(x) = 4$ and $\lim_{x \rightarrow 6^+} f(x) = 3$
 (D) $\lim_{x \rightarrow 3^-} f(x) = 3$ and $\lim_{x \rightarrow 4} f(x) = 4$ and $\lim_{x \rightarrow 6^+} f(x)$ does not exist
 (E) None of the above.

6. Suppose that $f(x) = \begin{cases} x - 14, & \text{if } x \leq 3 \\ Ax + B & \text{if } 3 < x < 9 \\ 2x - 41 & \text{if } x \geq 9. \end{cases}$

Find the values of A and B which make $f(x)$ continuous everywhere.

- (A) $A = 2, B = 9$
 (B) $A = -2, B = 5$
 (C) $A = 3, B = 9$
 (D) $A = -2, B = -5$
 (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

7. If $\lim_{x \rightarrow 2} f(x) = 3$ and $\lim_{x \rightarrow 2} g(x) = 5$, find $\lim_{x \rightarrow 2} \frac{g(x) - 5}{2f(x) - 5}$.

- (A) 1
- (B) 5
- (C) 0
- (D) 3
- (E) Limit does not exist

8. A function $f(x)$ is given by the formula $f(x) = A \cdot e^{kx}$ for constants A and k . We know that $f(5) = 8$ and $f(7) = 2$. Find the value of A and k .

- (A) $A = 2, k = \ln(7/5)$
- (B) $A = \frac{2}{e^{5 \ln(1/2)}}, k = 5 \ln(1/2)$
- (C) $A = \frac{8}{e^{5 \ln(2)}}, k = \ln(2)$
- (D) $A = \frac{8}{e^{5 \ln(1/2)}}, k = \ln(1/2)$
- (E) None of the above

9. If a function $f(x)$ is not defined at $x = a$, which of the following is a true statement?

- (A) $\lim_{x \rightarrow a} f(x)$ cannot exist
- (B) $\lim_{x \rightarrow a} f(x)$ might be equal to zero
- (C) $\lim_{x \rightarrow a} f(x)$ must approach ∞
- (D) $\lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a^-} f(x)$
- (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

10. Find a solution for $\ln(x^7) - \ln(x^2) = 15$.

- (A) $15^{1/5}$
- (B) e^5
- (C) e^3
- (D) e^{15/x^5}
- (E) None of the above

11. Why is the following function discontinuous at $x = 0$?

$$f(x) = \begin{cases} e^{2x} & \text{if } x < 0 \\ x^5 & \text{if } x \geq 0 \end{cases}$$

- (A) $f(0)$ does not exist
- (B) $\lim_{x \rightarrow 0} f(x)$ does not exist or is infinite
- (C) Both (A) and (B)
- (D) $f(0)$ and $\lim_{x \rightarrow 0} f(x)$ exist, but they are not equal
- (E) None of the above.

12. Which of the following functions is continuous at $x = 7$?

- (A) $\frac{1}{x^2 - 49}$
- (B) $\frac{e^{x/7}}{\cos(x - 7)}$
- (C) $\frac{x - 7}{x^2 - 49}$
- (D) $\frac{1}{\sin(x - 7)}$
- (E) None of the above

Free Response Questions: Show your work!

13. Assume that the position of an object after t seconds is given by $f(t) = 5t^2 + 2t$ meters.

(a) Write an expression for the average velocity of the object on the interval $[-1, t]$. Include units!

(b) Compute the average velocity over the time intervals $[-1, -0.999]$ and $[-1.001, -1]$ to estimate the instantaneous velocity. Include units!

(c) Take the limit as t approaches -1 of the expression you found in part (a) to find the instantaneous velocity of the object at time $t = -1$ seconds. Include units!

Free Response Questions: Show your work!

14. Find the limits or state that the limit does not exist. In each case, justify your answer.

(Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{x \rightarrow 1} \left[\frac{8}{1-x} - \frac{16}{1-x^2} \right]$

(b) $\lim_{x \rightarrow 0} x \cos(7/x)$

Free Response Questions: Show your work!

15. Find the limits or state that the limit does not exist. In each case, justify your answer.
(Students who guess the answer based on a few values of the function will not receive full credit.)

(a)
$$\lim_{x \rightarrow \infty} \frac{7x^5 - 3x^3 + \pi}{11x^5 + x^4 - x^2 + 2x}$$

(b)
$$\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 2}}{3x + 1}$$

Free Response Questions: Show your work!

16. (a) State the Intermediate Value Theorem.

(b) Let $f(x) = x^3 + 3x - 10$. Use the Intermediate Value Theorem to prove that $f(x) = 0$ has a solution in the interval $[1, 2]$.