Exam 1

Form A

Name: _____ Section and/or TA: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 12 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1	A	B	C	D	E	**7**	A	B	C	D	E
2	A	B	C	D	E	**8**	A	B	C	D	E
3	A	B	C	D	E	**9**	A	B	C	D	E
4	A	B	C	D	E	**10**	A	B	C	D	E
5	A	B	C	D	E	**11**	A	B	C	D	E
6	A	B	C	D	E	**12**	A	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Find the exact value of the expression

$$\log_5 100 + \log_5 25 - 2\log_5 2.$$

A. 4
B. 5
C. 6
D. 7
E. 8

2. If $f(x) = x + 5$ and $h(x) = 4x - 10$, find a function $g(x)$ so that $g(f(x)) = h(x)$.

A. $g(x) = 4x + 30$
B. $g(x) = 4x$
C. $g(x) = x - 30$
D. $g(x) = 4x - 30$
E. $g(x) = x + 30$

3. Find the inverse function of $f(x) = \dfrac{x+1}{4x+1}$.

A. $f^{-1}(x) = -\dfrac{4x+1}{x-1}$

B. $f^{-1}(x) = \dfrac{x}{4x-1}$

C. $f^{-1}(x) = \dfrac{4x+1}{x+1}$

D. $f^{-1}(x) = \dfrac{x+1}{\frac{1}{4}x+1}$

E. $f^{-1}(x) = -\dfrac{x-1}{4x-1}$

4. Evaluate the limit

$$\lim_{x\to 1} (x+5)^3 \left(x^2-6\right)$$

A. -1090

B. -1080

C. -1070

D. -448

E. 320

5. Given that $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = -4$, and $\lim_{x\to a} h(x) = 2$, find

$$\lim_{x\to a} \Big((h(x))^2 - f(x)g(x) \Big).$$

A. 16

B. 17

C. 22

D. -8

E. 0

6. If $1 \leq f(x) \leq x^2 + 2x + 2$, for all x, find $\lim_{x\to -1} f(x)$.

A. $-1/8$

B. $-1/16$

C. 1

D. 8

E. Does not exist

7. Simplify the following: $\sin(2\arctan(x)) = \sin(2\tan^{-1}(x))$.

   A. $\dfrac{2x}{1+x^2}$

   B. $\dfrac{2x}{\sqrt{1+x^2}}$

   C. $\dfrac{x}{\sqrt{1+x^2}}$

   D. $\dfrac{x}{1+x^2}$

   E. $\dfrac{2}{1+x^2}$

8. Find the equation of the line passing through the points $(-1,3)$ and $(2,9)$.

   A. $y = 2x+1$

   B. $y = 2x+4$

   C. $y = 2x+5$

   D. $y = \frac{1}{2}x + \frac{7}{2}$

   E. $y = \frac{1}{2}x + \frac{5}{2}$

9. Find $\arcsin\left(\sin\left(\frac{7\pi}{6}\right)\right)$.

A. $\frac{7\pi}{6}$

B. $-\frac{\pi}{6}$

C. $\frac{\pi}{6}$

D. $\frac{5\pi}{6}$

E. $-\frac{5\pi}{6}$

10. Solve the equation for x:

$$3^{x^2-3x} = 9^{x+7}$$

A. $x = -7$ and $x = 2$

B. $x = 2$ and $x = 7$

C. $x = -2$ and $x = 7$

D. $x = 2 \pm \sqrt{11}$

E. There is no solution.

11. The population of a city at time t is $P(t) = 500e^{.075t}$. When will the population be four times larger than $P(0)$?

    A. $\dfrac{\ln(0.075)}{4}$

    B. $\dfrac{\ln(4)}{0.075}$

    C. $0.075\ln(4)$

    D. $500\ln(4)$

    E. None of the above

12. A stone is tossed in the air from ground level. Its height at time t is $h(t) = 45t - 4.9t^2$ meters. Compute the average velocity of the stone over the time interval [1.5, 3.5].

    A. 41 m/s

    B. 30.3 m/s

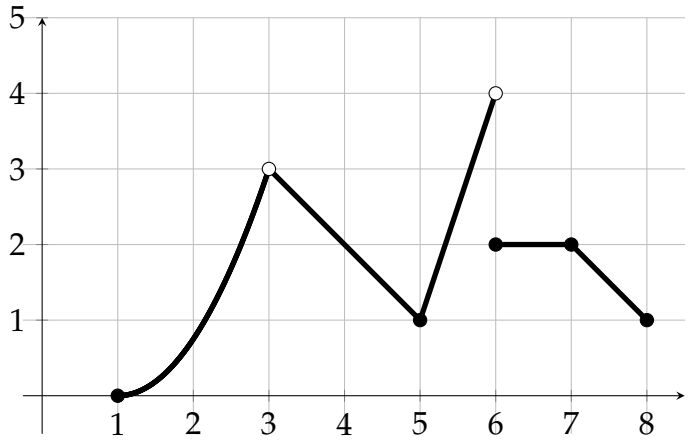
    C. 20.5 m/s

    D. 10.7 m/s

    E. None of the above

Free Response Questions

13. Given that $\tan(\theta) = \frac{5}{12}$ and $0 \leq \theta \leq \frac{\pi}{2}$, find $\sin(\theta)$, $\cos(\theta)$, $\sec(\theta)$, $\sin(2\theta)$ and $\cos(2\theta)$.



14. The graph of $f(x)$ is shown above. Find the following limits if they exist.

(a) $\lim_{x \to 6^-} f(x)$

(b) $\lim_{x \to 6^+} f(x)$

(c) $\lim_{x \to 6} f(x)$

(d) $\lim_{x \to 3} f(x)$

(e) $\lim_{x \to 5} f(x)$

15. Find the limits or state that the limit does not exist. In each case, justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{x\to 2} \dfrac{x^3 - 4x}{x - 2}$

(b) $\lim_{x\to 3} 3x - 4 + \dfrac{x^2}{x - 2}$

(c) $\lim_{x\to 2^+} f(x)$ if $f(x) = \begin{cases} x^2 - 2x + 2 & \text{if } x \leq 2 \\ -4x + 12 & \text{if } x > 2 \end{cases}$

16. Assume that the position of an object after t seconds is given by $f(t) = 10t^2 + 3t$ meters.

    (a) Write an expression for the average velocity on the interval $[2, 2+h]$. Include units!

    (b) Compute the average velocity over the time intervals $[1.999, 2]$ and $[2, 2.001]$ to estimate the instantaneous velocity. Include units!

    (c) Take the limit as h approaches 0 of the expression you found in part (a) to find the instantaneous velocity of the object at time $t = 2$ seconds. Include units!