Exam 1

Form A

Name: ______________________________ Section and/or TA: ________________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 16 multiple choice questions that count 4 points each and 4 free response questions that count 9 points each. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1	A	B	C	D	E	**9**	A	B	C	D	E
2	A	B	C	D	E	**10**	A	B	C	D	E
3	A	B	C	D	E	**11**	A	B	C	D	E
4	A	B	C	D	E	**12**	A	B	C	D	E
5	A	B	C	D	E	**13**	A	B	C	D	E
6	A	B	C	D	E	**14**	A	B	C	D	E
7	A	B	C	D	E	**15**	A	B	C	D	E
8	A	B	C	D	E	**16**	A	B	C	D	E

SCORE

Multiple Choice	17	18	19	20	Total Score
64	9	9	9	9	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Which of the following pairs of f and g has the composition $f \circ g(x) = 9x - 21$?

   A. $f(x) = 3x - 5$ and $g(x) = 3x - 6$

   B. $f(x) = 3x - 6$ **and** $g(x) = 3x - 5$

   C. $f(x) = 3x - 5$ and $g(x) = 3x - 7$

   D. $f(x) = 3x - 7$ and $g(x) = 3x - 5$

   E. $f(x) = 3x - 5$ and $g(x) = 3x - 5$

2. Find the inverse of the function $f(x) = 2021^{9x-21}$.

   A. $f^{-1}(x) = \frac{1}{9}\log_{2021}(x + 21)$

   B. $f^{-1}(x) = \log_{2021}(9x - 21)$

   C. $f^{-1}(x) = \log_{2021}(9x) - 21$

   D. $f^{-1}(x) = \frac{1}{9}\left(\log_{2021} x + 21\right)$

   E. $f^{-1}(x) = \log_{2021}\left(\frac{1}{9}x + 21\right)$

3. Find the domain of $f(x) = \sqrt{\ln(9x - 21)} + 2021$.

   A. $x \geq -\dfrac{20}{9}$

   B. $x \geq \dfrac{21}{9}$

   C. $x \geq \dfrac{22}{9}$

   D. $x \geq \dfrac{e+21}{9}$

   E. $x \geq e^{\sqrt{\ln(9x-21)}} - 2021$

4. Given that $\tan(\theta) = \frac{4}{3}$ and $0 \leq \theta \leq \frac{\pi}{2}$, find $\cos(\theta)$.

   A. $-\dfrac{4}{5}$

   B. $-\dfrac{3}{5}$

   C. $\dfrac{3}{5}$

   D. $\dfrac{4}{5}$

   E. None of the above

5. Find a value of c so that the function

$$f(x) = \begin{cases} x^2 + 2x + c & x \leq 2 \\ 2cx - 1 & x > 2 \end{cases}$$

is continuous for all x

A. $c = -4$

B. $c = -\frac{9}{5}$

C. $c = -\frac{1}{4}$

D. $c = 2$

E. $c = 3$

6. Which is the horizontal asymptote of the function $f(x) = \dfrac{e^x + 3e^{-2x} + 1}{2e^x}$?

A. $y = -\infty$

B. $y = 0$

C. $y = 0.5$

D. $y = 1$

E. $y = \infty$

7. If the displacement (in meters) of a particle moving in a straight line is given by $s(t) = t^2 - 2t + 1$ where t is measured in seconds, then the instantaneous velocity when $t = 3$ is

   A. 1 meters per second

   B. 2 meters per second

   C. 3 meters per second

   D. 4 meters per second

   E. 6 meters per second

8. Evaluate the limit:

$$\lim_{x\to\infty} \frac{\sqrt[3]{27x^3 + 2x^2 + \sin x}}{4x + 3}$$

   A. 0.5

   B. 0.75

   C. 6.25

   D. ∞

   E. Does not exist.

9. For $-1 \leq x \leq 1$ find $\tan(\arcsin(x)) = \tan(\sin^{-1}(x))$.

   A. $\dfrac{1}{\sqrt{1-x^2}}$

   B. $\dfrac{1}{\sqrt{1+x^2}}$

   C. $\dfrac{x}{\sqrt{1-x^2}}$

   D. $\dfrac{x}{\sqrt{1+x^2}}$

   E. $\dfrac{x}{1-x^2}$

10. Suppose that an object moves along the y-axis so that its location is $y = x^2 + 5x$ at time x. (Here y is in meters and x is in seconds.) Find the average velocity for x changing from 4 to $4 + h$ seconds.

    A. $1 + h$

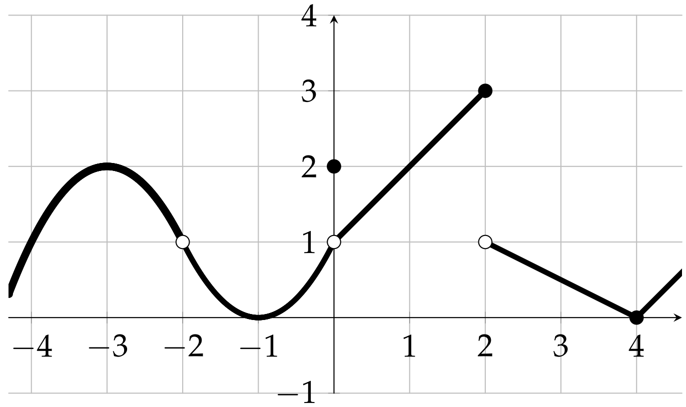
    B. $5 + h$

    C. $9 + h$

    D. 13

    E. $13 + h$

11. For the function graphed below, at how many places does the limit not exist.



A. 0

B. 1

C. 2

D. 3

E. 4

12. The vertical asymptotes of the function

$$f(x) = \frac{x^3 + x}{x^3 - x}$$

are

A. $x = -1$ and $x = 1$

B. $x = 0$ and $x = -1$

C. $x = 0$ and $x = 1$

D. $x = 0$, $x = -1$, and $x = 1$

E. This function has no vertical asymptotes.

13. Let $f(x) = x^3 + 2x$. For one of the answers below, you may use the Intermediate Value Theorem to find a solution to the given equation in the given interval. Select this answer.

   A. $f(x) = -5$, $(-2, -1)$

   B. $f(x) = -1$, $(-2, -1)$

   C. $f(x) = 0$, $(1, 2)$

   D. $f(x) = 0$, $(-2, -1)$

   E. $f(x) = 2$, $(1, 2)$

14. Suppose that f is defined on the real line and satisfies $3 \leq f(x) \leq 3 + (x-1)^2$. There is one value a where we may use the Squeeze Theorem to show that the limit $\lim_{x \to a} f(x) = L$ exists and has the value L. Find a and L.

   A. $a = 0$, $L = 4$

   B. $a = 1$, $L = 3$

   C. $a = 3$, $L = 1$

   D. $a = 3$, $L = 3$

   E. $a = 3$, $L = 7$

15. Suppose that $\lim_{x \to 3} f(x) = 7$ and $\lim_{x \to 3} g(x) = 2$, find $\lim_{x \to 3} \dfrac{x + f(x)}{f(x) - 3g(x)}$.

A. 7

B. 8

C. 9

D. 10

E. The limit does not exist.

16. Select the correct statement about the function $f(x) = \dfrac{x^3 - 8}{x - 2}$.

A. $f(2) = 12$

B. The function is continuous at $x = 2$.

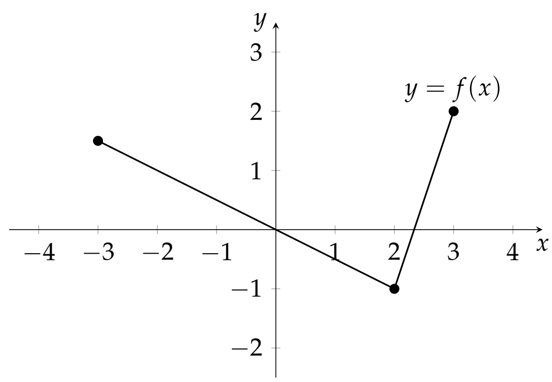
C. The function has a removable discontinuity at $x = 2$.

D. The function has an infinite discontinuity at $x = 2$.

E. The function has a jump discontinuity at $x = 2$.

Free Response Questions
Show all of your work

17. Consider the function $f(x)$ found in the graph below.



(a) What is the domain of $f(x)$?

Solution: Since the function is defined for $-3 \leq x \leq 3$ the domain of the function is $[-3, 3]$.

(b) What is the range of $f(x)$?

Solution: The values of the function are between -1 and 2 so the range of the function is $[-1, 2]$.

(c) Does the function $f(x)$ have an inverse? Why?

Solution: The function does not have an inverse because it is not one-to-one. It does not pass the horizontal line test.

(d) Find the formula for $f(x)$ on $2 \leq x \leq 3$.

Solution: $f(2) = -1$ and $f(3) = 2$ and the graph of f is the line segment between those two points. This means that in order to describe f between these two points, we need the equation of the line between the two points. The slope of the line is:

$$\text{slope} = m = \frac{2 - (-1)}{3 - 2} = 3.$$

The equation of the line is then given by $y = -1 + 3(x - 2) = 3x - 7$, so a formula for $f(x)$ on the interval $[2, 3]$ is $f(x) = 3x - 7$.

18. Find $f(x)$ and a such that $f'(a) = \lim_{h\to 0} \dfrac{\tan(\frac{\pi}{4}+h)-1}{h}$.

Solution: The derivative of a function $f(x)$ at $x = a$ is given by the limit

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$

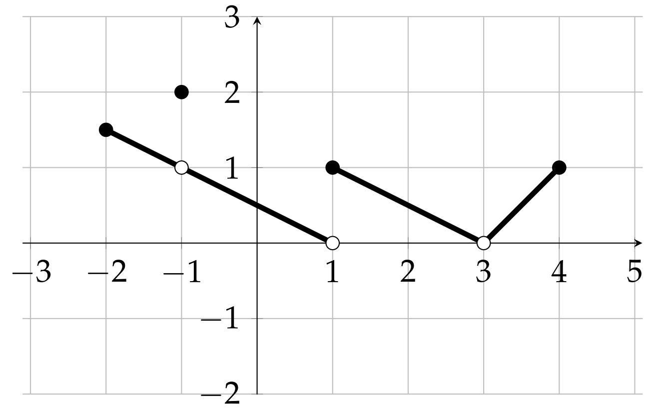
We need to match the given limit to a limit of this form for a given a and a particular $f(x)$. Note that it would appear as if $a = \dfrac{\pi}{4}$ and $f(x)$ would appear to be $\tan(x)$. To verify this we need to check the value of $\tan(x)$ at $x = \frac{\pi}{4}$ is the value being subtracted from $\tan(\frac{\pi}{4}+h)$. Sure enough, $\tan(\frac{\pi}{4}) = 1$ and this tells us that

$f(x) =$ $\underline{\quad\tan(x)\quad}$

and

$a =$ $\underline{\quad\pi/4\quad}$

19. Let f be the function whose graph is shown below. Evaluate each of the following expressions. If a limit does not exist or is undefined, answer "DNE".



(a) $\lim_{x \to -1^-} f(x) =$ 1

(b) $\lim_{x \to -1^+} f(x) =$ 1

(c) $\lim_{x \to -1} f(x) =$ 1

(d) $f(-1) =$ 2

(e) $\lim_{x \to 1^-} f(x) =$ 0

(f) $\lim_{x \to 1^+} f(x) =$ 1

(g) $\lim_{x \to 1} f(x) =$ does not exist

(h) $\lim_{x \to 3} f(x) =$ 0

(i) $f(3) =$ is not defined

20. Explain why each of the limits below exists or does not exist. Give the value for the limits that exist. Students who guess the value using a table of function values will not receive full credit.

(a) $\lim_{x \to 2} \frac{x+1}{x^2+2x}$.

Solution: This is a rational function whose denominator is not zero at $x = 2$ so by the Substitution Theorem we find the limit by

$$\lim_{x \to 2} \frac{x+1}{x^2+2x} = \frac{2+1}{2^2+2 \cdot 2} = \frac{3}{8}.$$

(b) $\lim_{x \to 3} \frac{x^2-3x}{x-3}$

Solution: This rational function has both the numerator and denominator equal to zero at $x = 3$, so we need to modify it form.

$$\lim_{x \to 3} \frac{x^2-3x}{x-3} = \lim_{x \to 3} \frac{x(x-3)}{x-3} = \lim_{x \to 3} x = 3.$$

(c) $\lim_{x \to 0} \frac{|x|}{x}$

Solution: Since the absolute value of x is defined in two pieces

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

we need to consider left and right hand limits as x approaches 0.

$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = \lim_{x \to 0^-} -1 = -1$$
$$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$$

Since the left and right hand limits do not agree, the limit $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.