Exam 1

Form A

Name: ______________________ Section and/or TA: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 16 multiple choice questions that count 4 points each and 4 free response questions that count 9 points each. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

SCORE

Multiple Choice	17	18	19	20	Total Score
64	9	9	9	9	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Which of the following pairs of f and g has the composition $f \circ g(x) = 9x - 21$?

   A. $f(x) = 3x - 5$ and $g(x) = 3x - 6$

   B. $f(x) = 3x - 6$ and $g(x) = 3x - 5$

   C. $f(x) = 3x - 5$ and $g(x) = 3x - 7$

   D. $f(x) = 3x - 7$ and $g(x) = 3x - 5$

   E. $f(x) = 3x - 5$ and $g(x) = 3x - 5$

2. Find the inverse of the function $f(x) = 2021^{9x-21}$.

   A. $f^{-1}(x) = \frac{1}{9} \log_{2021}(x + 21)$

   B. $f^{-1}(x) = \log_{2021}(9x - 21)$

   C. $f^{-1}(x) = \log_{2021}(9x) - 21$

   D. $f^{-1}(x) = \frac{1}{9}\left(\log_{2021} x + 21\right)$

   E. $f^{-1}(x) = \log_{2021}\left(\frac{1}{9}x + 21\right)$

3. Find the domain of $f(x) = \sqrt{\ln(9x-21)} + 2021$.

   A. $x \geq -\dfrac{20}{9}$

   B. $x \geq \dfrac{21}{9}$

   C. $x \geq \dfrac{22}{9}$

   D. $x \geq \dfrac{e+21}{9}$

   E. $x \geq e^{\sqrt{\ln(9x-21)}} - 2021$

4. Given that $\tan(\theta) = \frac{4}{3}$ and $0 \leq \theta \leq \frac{\pi}{2}$, find $\cos(\theta)$.

   A. $-\dfrac{4}{5}$

   B. $-\dfrac{3}{5}$

   C. $\dfrac{3}{5}$

   D. $\dfrac{4}{5}$

   E. None of the above

5. Find a value of c so that the function

$$f(x) = \begin{cases} x^2 + 2x + c & x \leq 2 \\ 2cx - 1 & x > 2 \end{cases}$$

is continuous for all x

A. $c = -4$

B. $c = -\frac{9}{5}$

C. $c = -\frac{1}{4}$

D. $c = 2$

E. $c = 3$

6. Which is the horizontal asymptote of the function $f(x) = \dfrac{e^x + 3e^{-2x} + 1}{2e^x}$?

A. $y = -\infty$

B. $y = 0$

C. $y = 0.5$

D. $y = 1$

E. $y = \infty$

7. If the displacement (in meters) of a particle moving in a straight line is given by $s(t) = t^2 - 2t + 1$ where t is measured in seconds, then the instantaneous velocity when $t = 3$ is

   A. 1 meters per second

   B. 2 meters per second

   C. 3 meters per second

   D. 4 meters per second

   E. 6 meters per second

8. Evaluate the limit:

$$\lim_{x\to\infty} \frac{\sqrt[3]{27x^3 + 2x^2 + \sin x}}{4x + 3}$$

   A. 0.5

   B. 0.75

   C. 6.25

   D. ∞

   E. Does not exist.

9. For $-1 \leq x \leq 1$ find $\tan(\arcsin(x)) = \tan(\sin^{-1}(x))$.

   A. $\dfrac{1}{\sqrt{1-x^2}}$

   B. $\dfrac{1}{\sqrt{1+x^2}}$

   C. $\dfrac{x}{\sqrt{1-x^2}}$

   D. $\dfrac{x}{\sqrt{1+x^2}}$

   E. $\dfrac{x}{1-x^2}$

10. Suppose that an object moves along the y-axis so that its location is $y = x^2 + 5x$ at time x. (Here y is in meters and x is in seconds.) Find the average velocity for x changing from 4 to $4+h$ seconds.

    A. $1+h$

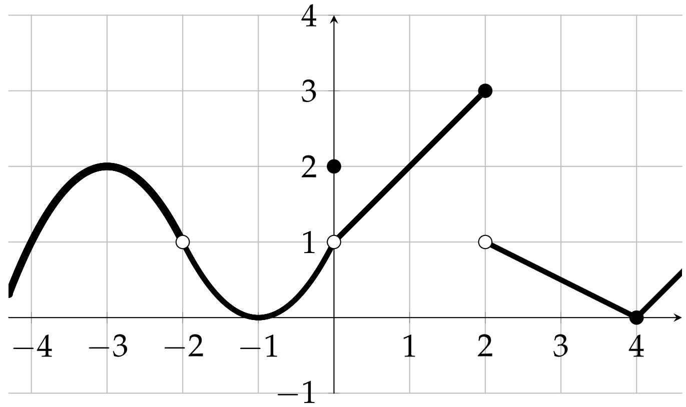
    B. $5+h$

    C. $9+h$

    D. 13

    E. $13+h$

11. For the function graphed below, at how many places does the limit not exist.



A. 0
B. 1
C. 2
D. 3
E. 4

12. The vertical asymptotes of the function

$$f(x) = \frac{x^3 + x}{x^3 - x}$$

are

A. $x = -1$ and $x = 1$
B. $x = 0$ and $x = -1$
C. $x = 0$ and $x = 1$
D. $x = 0$, $x = -1$, and $x = 1$
E. This function has no vertical asymptotes.

13. Let $f(x) = x^3 + 2x$. For one of the answers below, you may use the Intermediate Value Theorem to find a solution to the given equation in the given interval. Select this answer.

A. $f(x) = -5$, $(-2, -1)$

B. $f(x) = -1$, $(-2, -1)$

C. $f(x) = 0$, $(1, 2)$

D. $f(x) = 0$, $(-2, -1)$

E. $f(x) = 2$, $(1, 2)$

14. Suppose that f is defined on the real line and satisfies $3 \leq f(x) \leq 3 + (x-1)^2$. There is one value a where we may use the Squeeze Theorem to show that the limit $\lim_{x \to a} f(x) = L$ exists and has the value L. Find a and L.

A. $a = 0$, $L = 4$

B. $a = 1$, $L = 3$

C. $a = 3$, $L = 1$

D. $a = 3$, $L = 3$

E. $a = 3$, $L = 7$

15. Suppose that $\lim_{x\to 3} f(x) = 7$ and $\lim_{x\to 3} g(x) = 2$, find $\lim_{x\to 3} \frac{x + f(x)}{f(x) - 3g(x)}$.

A. 7

B. 8

C. 9

D. 10

E. The limit does not exist.

16. Select the correct statement about the function $f(x) = \frac{x^3 - 8}{x - 2}$.

A. $f(2) = 12$

B. The function is continuous at $x = 2$.

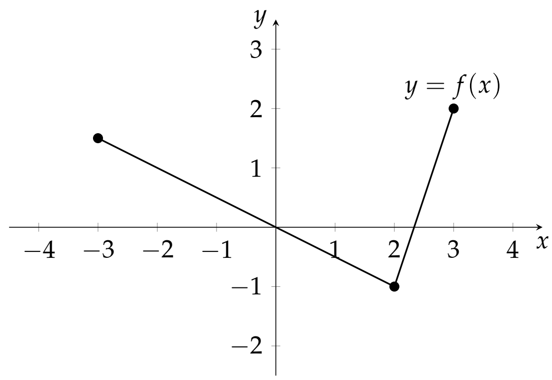
C. The function has a removable discontinuity at $x = 2$.

D. The function has an infinite discontinuity at $x = 2$.

E. The function has a jump discontinuity at $x = 2$.

Free Response Questions
Show all of your work

17. Consider the function $f(x)$ found in the graph below.



(a) What is the domain of $f(x)$?

(b) What is the range of $f(x)$?

(c) Does the function $f(x)$ have an inverse? Why?

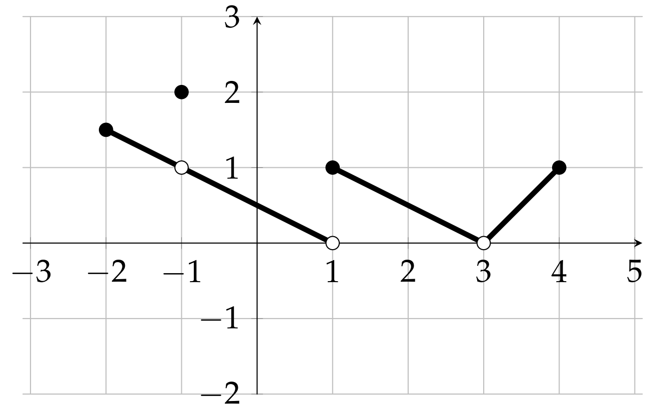
(d) Find the formula for $f(x)$ on $2 \le x \le 3$.

18. Find $f(x)$ and a such that $f'(a) = \lim_{h\to 0} \frac{\tan(\frac{\pi}{4}+h)-1}{h}$.

$f(x) =$ ____________________

$a =$ ____________________

19. Let f be the function whose graph is shown below. Evaluate each of the following expressions. If a limit does not exist or is undefined, answer "DNE".



(a) $\lim_{x\to -1^-} f(x) =$ ________________

(b) $\lim_{x\to -1^+} f(x) =$ ________________

(c) $\lim_{x\to -1} f(x) =$ ________________

(d) $f(-1) =$ ________________

(e) $\lim_{x\to 1^-} f(x) =$ ________________

(f) $\lim_{x\to 1^+} f(x) =$ ________________

(g) $\lim_{x\to 1} f(x) =$ ________________

(h) $\lim_{x\to 3} f(x) =$ ________________

(i) $f(3) =$ ________________

20. Explain why each of the limits below exists or does not exist. Give the value for the limits that exist. Students who guess the value using a table of function values will not receive full credit.

(a) $\lim\limits_{x \to 2} \dfrac{x+1}{x^2+2x}$.

(b) $\lim\limits_{x \to 3} \dfrac{x^2-3x}{x-3}$

(c) $\lim\limits_{x \to 0} \dfrac{|x|}{x}$