Name: ______________________ Section and/or TA: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that count 5 points each and 4 free response questions that count 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **11** (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E) **12** (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Give the domain of the function $f(x) = \dfrac{x^2+1}{x^2-9}$.

   A. $(-\infty, 3) \cup (3, \infty)$

   B. $(-\infty, -9) \cup (-9, 9) \cup (9, \infty)$

   C. $(-\infty, 9) \cup (9, \infty)$

   D. $(-\infty, -1) \cup (-1, 9) \cup (9, \infty)$

   E. $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

2. (5 points) Let $f(x) = (x-1)^2$ with the domain $(-\infty, 1]$ and let f^{-1} be the inverse function. Find $f^{-1}(5)$.

   A. 2

   B. 1/16

   C. $1 - \sqrt{5}$

   D. $1 + \sqrt{5}$

   E. -2

3. (5 points) A ball is thrown in the air so that after t seconds its height is $h(t) = 100t - 5t^2$ meters above the ground. Give the average velocity on the interval $[2, 2.4]$

   A. 80 meters/second

   B. 78 meters/second

   C. 31.2 meters/second

   D. 34 meters/second

   E. 32.4 meters/second

4. (5 points) Use the graph at right to find the one-sided limit $\lim_{x \to 1^-} f(x)$.

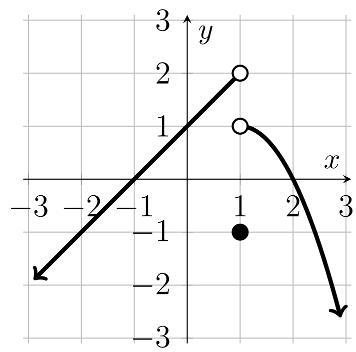
   A. -2

   B. 2

   C. -1

   D. 1

   E. The limit does not exist



5. (5 points) If $\lim_{x\to -1} f(x) = a$ and $\lim_{x\to -1} \dfrac{f(x)+2}{x+3} = 2$, find a.

   A. $a = 2$

   B. $a = 0$

   C. $a = 1$

   D. $a = 3$

   E. $a = 4$

6. (5 points) Find the limit $\lim_{s\to\infty} \sqrt{s} - \sqrt{s-4}$.

   A. $-\infty$

   B. 1

   C. 0

   D. -1

   E. ∞

7. (5 points) Consider the function g defined by

$$g(x) = \begin{cases} ax + 2, & \text{if } x < 2 \\ 2 - 2x & \text{if } x \geq 2, \end{cases}.$$

For which value of a is the function g continuous?

A. $a = -1$

B. $a = -2$

C. $a = 1$

D. $a = 2$

E. This function is not continuous for any value of a.

8. (5 points) Let $f(x) = \dfrac{(x-1)^2}{(x^2+2x-3)(x+2)}$. Select the set on which f is continuous.

A. $(-\infty, -3) \cup (-3, -2) \cup (-2, 1) \cup (1, \infty)$

B. $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

C. $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

D. $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

E. $(-\infty, -3) \cup (-3, -2) \cup (-2, 2) \cup (2, \infty)$

9. (5 points) Find the limit $\lim_{x \to -\infty} \frac{5 - 4x^5}{6 + 2x^4}$.

   A. 0

   B. -2

   C. $+\infty$

   D. $-\infty$

   E. 2

10. (5 points) Suppose the equation of the tangent line of f at $x = 2$ is $y = 4x - 5$. Select the correct statement.

   A. $f(2) = 4$

   B. $f(2) = 3$

   C. $f'(2) = -5$

   D. $f(2) = -5$

   E. $f'(2) = 1$

11. (5 points) Consider the limit $\lim_{h \to 0} \frac{\cos(2+h) - \cos(2)}{h} = L$. Select the correct statement.

A. The value L is the derivative of $\cos(x)$ at $x = 0$.

B. The value L is the derivative of $\cos(2 + x)$ at $x = 2$.

C. The value L is the derivative of $\cos(2x)$ at $x = 1$.

D. The value L is the derivative of $\cos(x)$ at $x = 2$.

E. The value L is the derivative of $\cos(x)$ at $x = h$.

12. (5 points) The graph of f is shown below. For which x is the derivative $f'(x) = 0$?

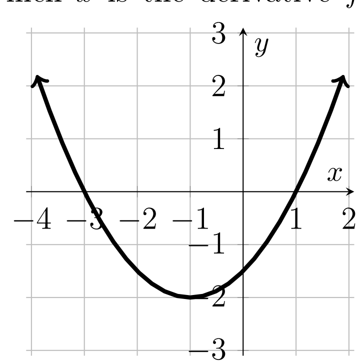
A. $x = -2$

B. $x = -3$

C. $x = -1$

D. $x = 0$

E. $x = 1$



Free response questions, show all work

13. (10 points) Let $f(x) = \dfrac{2x+5}{3x-4}$.
    (a) Find the formula for the inverse function f^{-1}.
    (b) Give the domain and range of f.
    (c) Give the domain and range of f^{-1}.

Solution: a) We write $y = \frac{2x+5}{3x-4}$ and solve to express x in terms of y.

$$\begin{aligned} y &= \frac{2x+5}{3x-4} \\ (3x-4)y &= 2x+5 \\ 3xy - 4y - 2x &= 5 \\ (3y-2)x &= 4y+5 \\ x &= \frac{4y+5}{3y-2} \end{aligned}$$

Thus, we have $f^{-1}(y) = \frac{4y+5}{3y-2}$. Replacing y by x gives $f^{-1}(x) = \dfrac{4x+5}{3x-2}$.

b,c) From the formula defining f, we see that the domain of f is

$$\{x : x \neq 4/3\} = (-\infty, 4/3) \cup (4/3, \infty)$$

From the formula defining f^{-1}, we see that the domain of f^{-1} is

$$\{x : x \neq 2/3\} = (-\infty, 2/3) \cup (2/3, \infty).$$

Since the range of f is domain of f^{-1} and the range of f^{-1} is domain of f, we have

Function	Domain	Range
f	$(-\infty, 4/3) \cup (4/3, \infty)$	$(-\infty, 2/3) \cup (2/3, \infty)$
f^{-1}	$(-\infty, 2/3) \cup (2/3, \infty)$	$(-\infty, 4/3) \cup (4/3, \infty)$

Alternate approach: Divide to write $f(x) = \dfrac{2}{3} + \dfrac{23}{3(3x-4)}$ and recognize that the asymptote $y = 2/3$ is not in the range of f.

Grading: a) 2 points for method (solving for x in terms of y), 3 points for completing the algebra and giving answer (give 2 of 3 points for correct, except minor mistakes and 1 for some progress)

bc) 2 points for first domain or range and 1 point for each additional point (5 total).

Follow through–if the formula for f^{-1} is incorrect, but they give domain of the function they found, then award points in parts b) or c).

14. (10 points) For each limit, find the limit or state that it does not exist. Justify each answer.

(a) $\lim_{x\to 2} \frac{x^2 - x - 2}{x^2 - 4}$

(b) $\lim_{x\to -2} \frac{x^2 - x - 2}{x^2 - 4}$

(c) $\lim_{x\to 0} \frac{x^2 - x - 2}{x^2 - 4}$

Solution: 14 a)

$$\lim_{x\to 2} \frac{x^2 - x - 2}{x^2 - 4} = \lim_{x\to 2} \frac{(x-2)(x+1)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{(x+1)}{(x+2)} = \frac{2+1}{2+2} = \frac{3}{4}$$

-1 point if drop limit in equalities or keep writing limit after evaluation.

-1 point if one writes

$$\frac{(x-2)(x+1)}{(x-2)(x+2)} = \frac{(x+1)}{(x+2)}$$

without specifying $x \neq 2$.

14 b) Via the same simplification as part a we have

$$\lim_{x\to -2} \frac{x^2 - x - 2}{x^2 - 4} = \lim_{x\to -2} \frac{(x+1)}{(x+2)}$$

The right-hand and left-hand limits differ as x approaches -2, so the limit DNE:

$$\lim_{x\to -2+} \frac{(x+1)}{(x+2)} = -\infty$$

$$\lim_{x\to -2-} \frac{(x+1)}{(x+2)} = +\infty$$

The easiest way to see this is to write

$$\frac{x+1}{x+2} = \frac{x+1+1-1}{x+2} = \frac{x+2}{x+2} + \frac{-1}{x+2} = 1 + \frac{-1}{x+2}$$

A graph of this function will show the asymptotes at $x = -2$ and $y = 1$.

14c) Via the quotient rule for limits, we have

$$\lim_{x\to 0} \frac{x^2 - x - 2}{x^2 - 4} = \frac{\lim_{x\to 0}(x+1)}{\lim_{x\to 0}(x+2)} = \frac{0^2 - 0 - 2}{0^2 - 4} = 1/2.$$

Since polynomials are continuous everywhere and the denominator is continuous at $x = 0$, so we can just evaluate the numerator and denominator at $x = 0$.

Final answer: 1 point. Justification: 2 points.

-1 point if used factorization in part a, but no justification when substituting $x = 0$.

15. (10 points) (a) State the intermediate value theorem.

(b) Use the intermediate value theorem to show $x^3 - x^2 + 10 = 0$ has a solution. Be sure to give the interval on which you are applying the intermediate value theorem.

Solution: a) Theorem: If f is continuous on the interval $[a, b]$ and M lies between $f(a)$ and $f(b)$, then there is a value $c \in [a, b]$ so that $f(c) = M$.

b) Let $f(x) = x^3 - x^2 + 10$. Since f is a polynomial function, it is continuous everywhere. We try a few values. For example $f(0) = 10$, $f(-1) = 8$, and $f(-2) = -2$. Since $f(-1) > 0$ and $f(-2) < 0$ and f is continuous on the interval $[-2, -1]$, then there is a number c in $[-2, -1]$ so that $f(c) = 0$.

Grading: a) Hypotheses (2 points), conclusion (2 points), b) Give interval (1 point), show 0 lies between values at endpoints (3 points), observe function is continuous (2 points).

16. (10 points) Let $f(x) = \dfrac{1}{x+2}$. Use the limit definition of the derivative to find the derivative $f'(3)$.

Solution: We write the difference quotient and simplify

$$\begin{aligned}\frac{f(3+h)-f(3)}{h} &= \frac{1}{h}\cdot(\frac{1}{5+h}-\frac{1}{5}) \\ &= \frac{1}{h}\cdot\frac{5-(5+h)}{(5+h)\cdot 5} \quad \text{obtain common denominator} \\ &= \frac{-1}{25+5h}, \qquad \text{if } h \neq 0.\end{aligned}$$

Since the difference quotient and the simplified expression are equal if $h \neq 0$, they both have the same limit at 0. Using the rule for limit of sums and our basic limits, $\lim_{h\to 0}(25+5h) = 25$. Since this limit is not zero, we may use the reciprocal rule for limits to find

$$f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{-1}{25+5h} = -1/25.$$

Alternate approach: Use that $-1/(25+5h)$ is a rational function in h and 0 is in the domain to evaluate the limit.

Grading: Form difference quotient (3 points), simplify difference quotient to cancel h (2 points), find limit (3 points), provide reasoning for limit (2 points).