Name: ______________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **11** (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E) **12** (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Let f be defined by $f(x) = 3x + 6$, find the inverse function $f^{-1}(x)$.

   A. $f^{-1}(x) = \dfrac{1}{3x+6}$

   B. $f^{-1}(x) = \dfrac{1}{3}x - 6$

   C. $f^{-1}(x) = \dfrac{1}{3}x - 2$

   D. $f^{-1}(x) = -3x - 6$

   E. $f^{-1}(x) = 3x - 2$

Solution: Compare with WA1 or WW0.6 #2.

2. (5 points) A ball is thrown in the air so that its height h in meters at time t seconds is $h(t) = -5t^2 + 40t$. Find the average velocity on the interval [5,7].

   A. -20 meters/second

   B. 20 meters/second

   C. 35 meters/second

   D. -40 meters/second

   E. 40 meters/second

Solution: Compare WW1.1-2, #3, #5–8

3. (5 points) If $\lim_{x\to 3} f(x) = 5$, all but one of the following statements must be true. Select the statement that may be false.

   A. $\lim_{x\to 3} xf(x) = 15$

   B. $\lim_{x\to 3^-} f(x) = 5$

   C. $\lim_{x\to 3} f^2(x) = 25$

   D. $\lim_{x\to 3^+} f(x) = 5$

   E. $f(3) = 5$

Solution: The function value $f(3)$ may not exist or may be different from the limit so $f(3) = 5$ may be false.

Compare WW1.3-limits, #5 WA1.

4. (5 points) Which answer best describes the behavior of $f(x) = \dfrac{1}{x-2}$ at $x = 2$.

   A. $\lim_{x\to 2^+} \dfrac{1}{x-2} = -\infty$ and $\lim_{x\to 2^-} \dfrac{1}{x-2} = +\infty$

   B. $\lim_{x\to 2} \dfrac{1}{x-2} = -\infty$

   C. $f(2) = \infty$

   D. $\lim_{x\to 2^+} \dfrac{1}{x-2} = +\infty$ *and* $\lim_{x\to 2^-} \dfrac{1}{x-2} = -\infty$

   E. $\lim_{x\to 2} \dfrac{1}{x-2} = +\infty$

Solution: Compare WS1.3 #6.

5. (5 points) Suppose that $\lim_{x\to 3} f(x) = -2$. Find $\lim_{x\to 3}(xf(x) + x^2)$.

   A. 3   B. 0   C. 6   D. 13   E. 15

Solution: Compare WS 1.4, #3, WW1.4#2,3

6. (5 points) Find the limit $\lim_{x\to a} \dfrac{x^2 - a^2}{x - a}$ or state that it does not exist.

   A. $-2a$ B. $+\infty$ *C.* $2a$ D. 0   E. Does not exist

Solution: Compare WW 1.4 #4,7, WS 1.4 #6. Also exercises on finding derivative in §1.5.

To find the limit, simplify $\frac{x^2-a^2}{x-a} = x + a$. Then $\lim_{x\to a}(x + a) = 2a$.

7. (5 points) Let $a > 0$. Find the limit $\lim_{h \to 0} \dfrac{\sqrt{a+2h} - \sqrt{a}}{h}$.

A. $\dfrac{2}{\sqrt{2a}}$ *B.* $\dfrac{1}{\sqrt{a}}$ C. $\dfrac{2}{\sqrt{a}}$ D. $\dfrac{1}{\sqrt{2a}}$ E. $\dfrac{1}{2\sqrt{a}}$

Solution: Compare WS2.1-2, #3

8. (5 points) Suppose that $0 \leq f(x) \leq x^2 - 2x + 1$. There is one value a where we can use the squeeze theorem to find the limit $L = \lim_{x \to a} f(x)$. Find a and L.

A. $a = 1$, $L = 1$

B. $a = 1$, $L = 0$

C. $a = -1$, $L = 0$

D. $a = 0$, $L = 0$

E. $a = 0$, $L = 1$

Solution: Compare WW1.4 #10, WS1.4 #4,5

9. (5 points) Find $\lim\limits_{x\to-\infty} \dfrac{|x-2|}{x}$.

   A. $+\infty$ *B.* -1 C. $-\infty$ D. 1 E. 0

   Solution: Compare WW1.5 #8. Also WW1.5 #5.

10. (5 points) If the tangent line to the graph of f at $x = 2$ is $y = 3x - 5$, give the value of the derivative $f'(2)$.

    A. 2 B. 1 C. -5 D. 4 *E.* 3

    Solution: Compare WW2.3 #2,4,

11. (5 points) Give the largest set where the function $f(x) = \sqrt{x-1}$ is continuous

   A. $[1, \infty)$

   B. $[0, 1]$

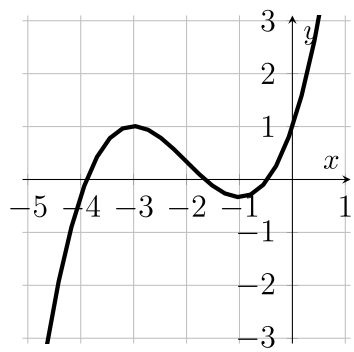
   C. $(-\infty, 1]$

   D. $(-\infty, 1) \cup (1, \infty)$

   E. $(-\infty, \infty)$

Solution: Compare WW1.6 #2. WS1.6, #3.

12. (5 points) The graph of f is shown at right. For which of the following values of x is the derivative $f'(x) < 0$?



A. $x = -1$ B. $x = -3$ C. $x = -2$ D. $x = -4$ E. $x = 0$

Solution: Compare WS2.3 #5, WW2.3 #6.

Free response questions, show all work

13. (10 points) For each of the following limits, give the value of the limit if the value is a finite number, $+\infty$, or $-\infty$ or state that the limit does not exist. Use the results about limits discussed in class to justify your answers.

(a) $\displaystyle\lim_{x\to 1} \frac{x^2-3x+2}{x-2}$

(b) $\displaystyle\lim_{x\to 2} \frac{x^2-3x+2}{x-2}$

(c) $\displaystyle\lim_{x\to 3} \frac{x^2-3x+2}{x-3}$

Solution: a) The function $(x^2-3x+2)/(x-2)$ is continuous at $x=1$, so we may evaluate the limit by substituting $x=1$, $\displaystyle\lim_{x\to 1} \frac{x^2-3x+2}{x-2} = 0$.

b) The value $x=2$ is not in the domain of $(x^2-3x+2)/(x-2)$ so we may not evaluate the limit by substitution.

If we simplify, we have

$$\frac{x^2-3x+2}{x-2} = \frac{(x-2)(x-1)}{x-2} = x-1, \quad x \neq 2.$$

Since $(x^2-3x+2)/(x-2) = x-1$ for $x \neq 1$, both functions have the same limit at $x=2$. Since $x=2$ is in the domain of the polynomial function $x-1$ we may evaluate the limit of $x-1$ by substitution. Combining these observations gives:

$$\lim_{x\to 2} \frac{x^2-3x+2}{x-2} = \lim_{x\to 2}(x-1) = 1.$$

c) For $x>3$, but x near 3, x^2-3x+2 is close to 2 and $x-3$ is small and positive, so the quotient $\dfrac{x^2-3x+2}{x-3}$ will be large and positive.

For $x<3$, but x near 3, x^2-3x+2 is close to 2 and $x-3$ is small and negative, so the quotient $\dfrac{x^2-3x+2}{x-3}$ will be large and negative. Thus we have

$$\lim_{x\to 3^+} \frac{x^2-3x+2}{x-3} = +\infty, \quad \lim_{x\to 3^-} \frac{x^2-3x+2}{x-3} = -\infty.$$

Since the two one-sided limits are different, the (ordinary or two-sided) limit does not exist (and is not $\pm\infty$).

Grading: a) We may evaluate limit of a rational/continuous function by substitution. (1 point) Value of limit (2 points)

b) Simplification (1 point), evaluation by substitution (1 point), value of limit (2 points)

c) One sided limits are different (1 point), limit does not exist (2 points).

Allow students to graph of numerical evidence to find one-sided limits.

Compare WS1.4 #2, WW1.4 #4.

14. (10 points) Consider the function f defined by

$$f(x) = \begin{cases} x+2, & x < 2 \\ a, & x = 2 \\ bx, & 2 < x \end{cases}$$

where a and b are constants.

(a) Find the limits $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$. Your answers may depend on a or b.

(b) Find the values of a and b so that f is continuous for all x.

Solution: a) The one sided limts are $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} bx = 2b$ and $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x+2) = 4$. In each, the limit of a continous or polynomial function can be evaluated by substitution.

b) To obtain continuity, we need $\lim_{x \to 2} f(x)$ to exist and equal $f(2)$. This will happen if the two one-sided limits are equal, $4 = 2b$ or $b = 2$. If $b = 2$, $\lim_{x \to 2} f(x) = 4$.

We will have $\lim_{x \to 2} f(x) = f(2)$ if $4 = a$.

Thus the values are $a = 4$ and $b = 2$.

a) Value of each limit (2 points each).

b) Equality of one-sided limits $4 = 2b$ (2 points), equality of limit and function value $a = 4$ or $a = 2b$ (2 points). Values of a and b (1 point each).

Compare WW1.6 #6

15. (10 points) (a) State the intermediate value theorem.

(b) Find an interval $[a, b]$ which contains a solution of the equation $x^3 + x = 3$. Use the intermediate value theorem to show that the interval you found contains a solution to this equation.

Solution: a) If f is a function that is continuous on a closed interval $[a, b]$ and L is a value between $f(a)$ and $f(b)$, then there is a value c in the interval $[a, b]$ so that $f(c) = L$.

b) The function $f(x) = x^3 + x$ is continuous since it is a polynomial. We consider the interval $[1, 2]$. Since $f(1) = 2 < 3$ and $f(2) = 10 > 3$, we may use the interval value theorem to conclude that there is a value c in $[1, 2]$ so that $f(c) = 3$.

Grading: a) f continuous (1 point), L between $f(a)$ and $f(b)$ (1 point), existence of a solution to $f(c) = L$ (1 point), c is in the interval $[a, b]$ (1 point).

b) Correct interval $[a, b]$, (2 points), choosing a function and observing it is continuous (2 points), choosing L and verifying L is between $f(a)$ and $f(b)$ (2 points).

Compare WS1.6 #7,8.

16. (10 points) (a) State the definition of the derivative of a function f at a point a.

(b) Use the definition to find the derivative $f'(3)$ for the function $f(x) = x^2 + x$.

Solution: a) The derivative of a function f at a point a is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided the limit exists.

Also accept $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.

b) For the function $x^2 + x$, we begin by simplifying the difference quotient,

$$\frac{f(x) - f(3)}{x - 3} = \frac{x^2 + x - 12}{x - 3} = \frac{(x-3)(x+4)}{x - 3} = x + 4.$$

The limit is

$$f'(3) = \lim_{x \to 3} (x + 4) = 7.$$

Grading: a) either form difference quotient (2 points), take limit at correct point (1 point), require limit to exist (1 point).

b) form difference quotient (2 points), simplify (2 points), evaluate limit correctly and give derivative (2 points).

Student who do not use definition can receive 2 points for answer.

Compare WS2.1-2 #2.