

Answer all of the questions 1 – 7 and two of the questions 8 – 10. Please indicate which problem is not to be graded by crossing through its number on the table below.

Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),
2. give exact answers, rather than decimal approximations.

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Last four digits of student identification number: \_\_\_\_\_

Question	Score	Total
1		9
2		7
3		11
4		12
5		11
6		9
7		6
8		16
9		16
10		16
Free	3	3
		100

(1) (a) Using logarithms solve the identity  $5^{4x-10} = \frac{1}{25}$ . **Show your work!**

(b) Simplify the expression

$$\log_3(x) + \frac{1}{2} \log_3(5x) - \log_3(10 + x)$$

so that it is a single logarithm.

(a)  $x =$  \_\_\_\_\_

(b) Expression is \_\_\_\_\_

(2) Suppose that  $\lim_{x \rightarrow 3} f(x)$  exists and satisfies

$$\lim_{x \rightarrow 3} \left( \frac{1}{x+2} f(x) + 3x - 1 \right) = 14.$$

Use the limit laws to compute  $\lim_{x \rightarrow 3} f(x)$ . **Explain your reasoning.**

$$\lim_{x \rightarrow 3} f(x) = \underline{\hspace{10em}}$$

(3) Let  $f(x) = x^2 - 2x - 8$ .

(a) Find the maximal value for  $a$  such that  $f$  is one-to-one on the interval  $(-\infty, a]$ .

(b) For the value of  $a$  you found in part (a) find the inverse  $f^{-1}$  of the function  $f$  with the domain  $(-\infty, a]$ .

(a)  $a =$  \_\_\_\_\_

(b)  $f^{-1}(x) =$  \_\_\_\_\_

(4) A particle is traveling on a straight line so that its position after  $t$  seconds is given by  $s(t) = 40t - 16t^2$  centimeters.

(a) Find the average velocity of the particle during the time interval  $[1, 3]$ .

(b) Find the average velocity of the particle during the time interval  $[1, t]$ , where  $t > 1$ . Simplify your answer.

(c) Find the instantaneous velocity of the particle at time  $t = 1$ .

(a) average velocity on  $[1, 3]$  is \_\_\_\_\_ cm/sec

(b) average velocity on  $[1, t]$  is \_\_\_\_\_ cm/sec

(c) instantaneous velocity at time  $t = 1$  is \_\_\_\_\_ cm/sec

(5) Compute the following limits or show that they do not exist.

(a)  $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - x - 6} = \underline{\hspace{2cm}}$

(b)  $\lim_{x \rightarrow 3} \frac{x^2 + x - 8}{x^2 - 4x - 5} = \underline{\hspace{2cm}}$

(c)  $\lim_{x \rightarrow 3} \frac{x^2 + x + 5}{x^2 - 4x + 3} = \underline{\hspace{2cm}}$

(6) Let  $f$  be the function defined as

$$f(x) = \begin{cases} x^2 + \sqrt{2x - 6}, & \text{for } x \geq 5 \\ x^2 + x - 3, & \text{for } x < 5 \end{cases}$$

Show that the function  $f$  is continuous at  $x = 5$ . **Clearly explain your reasoning.**

(7) Let  $f$  be a function such that, for all real numbers  $x$ ,

$$2x^3 - 3x^2 - 2 \leq f(x) \leq x^4 - 2x^3 + 4x^2 - 6x.$$

Show that  $\lim_{x \rightarrow 1} f(x)$  exists and find its value. **As usual, justify your answer.**

$$\lim_{x \rightarrow 1} f(x) = \underline{\hspace{2cm}}$$



Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front page of the exam.

(8) (a) Define what it means for a function  $f$  to be continuous at  $a$ . Use complete sentences.

(b) Let  $b$  and  $c$  be numbers and

$$f(x) = \begin{cases} 2 - x^2 & \text{for } x < 1, \\ bx + c & \text{for } 1 \leq x \leq 3, \\ 10 - x & \text{for } x > 3. \end{cases}$$

Find

$$\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}, \quad \lim_{x \rightarrow 1^+} f(x) = \underline{\hspace{2cm}},$$

$$\lim_{x \rightarrow 3^-} f(x) = \underline{\hspace{2cm}}, \quad \lim_{x \rightarrow 3^+} f(x) = \underline{\hspace{2cm}}.$$

Some of your answers involve  $b$  and  $c$ .

(c) Find the values of  $b$  and  $c$  such that the function  $f$  in part (b) is continuous at all real numbers.

$$b = \underline{\hspace{2cm}} \quad c = \underline{\hspace{2cm}}$$

(9) (a) State the definition of the derivative of a function  $f$  at a number  $a$ . Use complete sentences.

(b) Find the slope of the secant line to the graph of the function

$$f(x) = x^2 - 3x$$

through the points  $(a, f(a))$  and  $(a+h, f(a+h))$ , where  $h \neq 0$ . Simplify your answer.

(c) Use part (b) to find the derivative  $f'(a)$ .

(d) Find the equation of the tangent line to the graph of  $f$  at  $a = 4$ . Put your answer into the form  $y = mx + b$ .

(b) Slope of secant line = \_\_\_\_\_

(c) Derivative  $f'(a) =$  \_\_\_\_\_

(d) Equation is \_\_\_\_\_

(10) (a) State the Intermediate Value Theorem. Use Complete Sentences.

(b) Find an interval  $[a, b]$  of length at most 2 such that the equation

$$4x^5 + 10x^3 + 3x = -1$$

has a solution in  $(a, b)$ . **Justify your answer using the Intermediate Value Theorem.**