

Record the correct answer to the following problems on the front page of this exam.

1. The tangent line to the graph of the function f at $(2, f(2))$ is $y = 2x - 1$. Give the values of $f(2)$ and $f'(2)$.

- (A) $f(2) = 2$ and $f'(2) = 2$.
- (B) $f(2) = 3$ and $f'(2) = 2$
- (C) $f(2) = 3$ and $f'(2) = -1$
- (D) $f(2) = -1$ and $f'(2) = 2$
- (E) $f(2) = 2$ and $f'(2) = 3$

2. Let $f(x) = 3 \sin(\pi x)$. Then $f^{(5)}(1)$, the fifth derivative of f evaluated at $x = 1$, is:

- (A) 243
- (B) $3\pi^5$
- (C) $-3\pi^5$
- (D) -243
- (E) $3\pi^4$

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3. Let y be implicitly defined by $\sin y + \cos y = x^3$. Then $\frac{dy}{dx}$ evaluated at the point $(x, y) = (-1, \pi)$ is:

- (A) -3
- (B) 0
- (C) 1
- (D) -1
- (E) $-\frac{3}{2}$

4. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. Suppose that when the radius is 3 cm, the radius is decreasing at a rate of 2 cm/s. Here cm/s is the abbreviation for centimeters per second. Find the rate of change of the volume with respect to time when the radius is 3 cm.

- (A) $72\pi \text{ cm}^3/\text{s}$
- (B) $-72\pi \text{ cm}^3/\text{s}$
- (C) $36\pi \text{ cm}^3/\text{s}$
- (D) $-36\pi \text{ cm}^3/\text{s}$
- (E) None of the above.

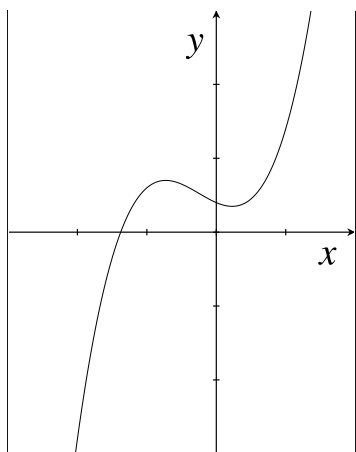
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5. Find the first derivative of $\arctan(e^{3x})$. The function \arctan is also denoted by \tan^{-1} .

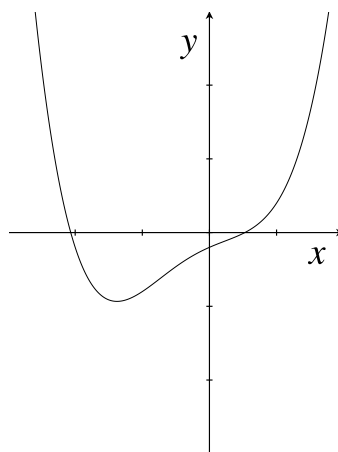
- (A) $3e^{3x} \sec^2(e^{3x})$
- (B) $\frac{3e^{3x}}{1 + e^{6x}}$
- (C) $-\frac{e^{3x}}{1 + e^{6x}}$
- (D) $\frac{1}{1 + e^{6x}}$
- (E) $\sec(e^{3x}) \cdot \tan(e^{3x})$

6. Below are graphs of a function f , its first derivative f' , and its second derivative f'' . Matching f and its derivatives with the correct graphs gives:

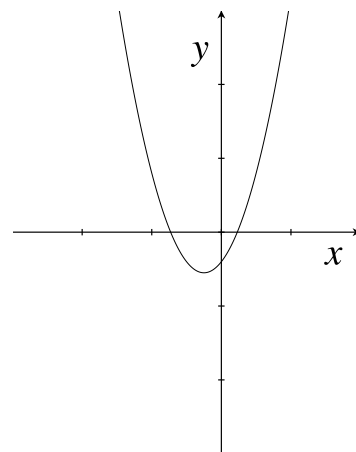
- (A) (a) is f , (b) is f' , and (c) is f'' .
- (B) (c) is f , (a) is f' , and (b) is f'' .
- (C) (a) is f , (c) is f' , and (b) is f'' .
- (D) (b) is f , (c) is f' , and (a) is f'' .
- (E) (b) is f , (a) is f' , and (c) is f'' .



(a)



(b)



(c)

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7. Find the equation of the tangent line to the graph of the function $f(x) = \frac{x^3}{\ln(x)}$ at the point $(e, f(e))$.

- (A) $y = 2e^2x - 2e$
- (B) $y = x - 3$
- (C) $y = 2e^2x - e^3$
- (D) $y = x - e^2 + 2e$
- (E) $y = e^3x + 4$

8. Suppose that $f(-1) = 0$, $f(0) = 1$, $f(1) = 2$, $f'(-1) = -2$, $f'(0) = -1$, and $f'(1) = -3$, and

$$g(x) = f\left(\arcsin\left(\frac{x}{2}\right) - 1\right).$$

The function \arcsin is also denoted by \sin^{-1} . The value of $g'(0)$ is

- (A) -2
- (B) -1
- (C) 0
- (D) 1
- (E) 2

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9. Let g and h be functions that are differentiable at 1 and let $f(x) = g(x) \cdot h(x)^2$. We are given that $g(1) = -1$, $h(1) = 2$, $g'(1) = 3$ and $h'(1) = 4$. Find $f'(1)$.

- (A) -1
- (B) -2
- (C) -3
- (D) -4
- (E) -5

10. Find the second derivative of $f(x) = e^{-2x^2}$.

- (A) $f''(x) = 16x^2 e^{-2x^2}$
- (B) $f''(x) = (16x^2 - 4) e^{-2x^2}$
- (C) $f''(x) = (4x^4 + 2x^2) e^{-2x^2-2}$
- (D) $f''(x) = (-4x + 16) e^{-2x^2}$
- (E) None of the above answers are correct.

Free Response Questions: Show your work!

11. Find the equation of the tangent line to the curve given by

$$x^2y^3 + 4y^2 + x = 25$$

at the point $(x, y) = (1, 2)$. Give the equation in the form $y = mx + b$.

Free Response Questions: Show your work!

12. (a) Give the definition of $f'(a)$, the derivative of a function f at the point a .

(b) Let f be the function defined by

$$f(x) = \begin{cases} ax + b, & x < 2 \\ x^2 + 1, & x \geq 2 \end{cases}$$

Use the definition of the derivative and properties of differentiable functions to find a and b so that f is differentiable at 2.

Free Response Questions: Show your work!

13. The height of an object falling with constant acceleration is given by a quadratic polynomial $h(t) = at^2 + bt + c$. An object is dropped from a building that is 64 meters tall with 0 initial velocity. The building is not on the Earth, but is on a planet where the acceleration of gravity is -8m/s^2 .

(a) Find a function $h(t)$ that gives the height of the object t seconds after it is dropped until it hits the ground.

(b) Find the time when the object hits the ground.

(c) Find the velocity of the object at the moment when it hits the ground.

Free Response Questions: Show your work!

14. At exactly 6:42 pm during the combine demolition derby on the fourth day of the County Fair, Combine A finds itself 30 meters south of the center of the fairgrounds while traveling north at a speed of 2 m/s, while at the same time Combine B is 40 meters west of the center of the fairgrounds and traveling east at a speed of 3 m/s.
- (a) Make a sketch to represent the given information.
- (b) Find the rate at which the distance between the combines is changing at exactly 6:42 pm. Be sure to explain your work. Use the sketch in part (a) to indicate the meaning of any variables you introduce in your solution.

Free Response Questions: Show your work!

15. (a) Write down a statement (theorem) in complete sentence form which gives a formula for computing the derivative of an inverse function $f^{-1}(x)$. Your statement should include conditions which guarantee that f has a differentiable inverse.

- (b) Let f be a function which is one to one and let f^{-1} be the inverse function. We are given that $f(0) = 1$, $f(1) = 0$, $f(2) = -1$, $f'(0) = 3$, $f'(1) = -2$, and $f'(2) = -1$. Find $f^{-1}'(0)$.