

1. Suppose $f(x) = (x^3 + 1)e^{-x}$. Find $f'(1)$.

- (A) $5e^{-1}$
- (B) $-5e^{-1}$
- (C) $-e^{-1}$
- (D) e^{-1}
- (E) $3e^{-1}$

2. Suppose $f(x) = x^3 - 2x + 5x^{-1}$. Find an equation of the tangent line to the graph of $y = f(x)$ at the point $(1, 4)$.

- (A) $y = -4x + 8$
- (B) $y = 4x$
- (C) $y = x + 3$
- (D) $y = 4$
- (E) $y = 3x + 1$

Record the correct answer to the following problems on the front page of this exam.

3. Suppose $f(x) = x \cos x + \sin x$. Find $f''(x)$.

- (A) $2 \cos x + x \sin x$
- (B) $-4 \cos x - x \sin x$
- (C) $-3 \sin x - x \cos x$
- (D) $-3 \sin x + x \cos x$
- (E) $-2 \cos x + 3x \sin x$

4. Suppose $y = \ln(9t^2 + 2t + 5)$. Find $y'(1)$.

- (A) $\frac{4}{5}$
- (B) $\frac{1}{2}$
- (C) $4 \ln 2$
- (D) $\frac{9}{16}$
- (E) $\frac{5}{4}$

Record the correct answer to the following problems on the front page of this exam.

5. Suppose $f(x) = \sin^2(x^3 + 1)$. Find $f'(x)$.

- (A) $x^3 \sin(x^3 + 1)$
- (B) $6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$
- (C) $2 \sin(x^3 + 1) \cos(x^3 + 1)$
- (D) $3x^5 \sin(x^3 + 1)$
- (E) $3x^2 \sin(x^3 + 1) \cos(3x^2)$.

6. Suppose that $y = f(x)$ and $y^3 x^4 - 10x + y = -3$. Find $\frac{dy}{dx}$ at $P = (2, 1)$.

- (A) $-\frac{11}{24}$
- (B) $\frac{15}{41}$
- (C) $-\frac{16}{25}$
- (D) $-\frac{21}{48}$
- (E) $-\frac{22}{49}$

Record the correct answer to the following problems on the front page of this exam.

7. Find $f'(x)$, where $f(x) = \sqrt{x^2 + \cos^2 x}$.

- (A) $\frac{1}{2}(x^2 + \cos^2 x)^{-1/2}$
- (B) $x(x^2 + \cos^2 x)^{-1/2}$
- (C) $(x^2 + \cos^2 x)^{-1/2}(x + \sin x \cos x)$
- (D) $(x^2 + \cos^2 x)^{-1/2}(x - \sin x \cos x)$
- (E) $(x^2 + \cos^2 x)^{1/2}(x + \sin x \cos x)$

8. Find $f'(x)$, if $f(x) = \frac{e^{3x}}{x+1}$.

- (A) $\frac{xe^{3x}}{(x+1)^2}$
- (B) $\frac{3xe^{3x}}{(x+1)^2}$
- (C) $\frac{(3x+2)e^{3x}}{(x+1)^2}$
- (D) $\frac{(x+3)e^{3x}}{(x+1)^2}$
- (E) $\frac{(3x+1)e^{3x}}{(x+1)^2}$

Record the correct answer to the following problems on the front page of this exam.

9. Find $g'(e)$, where $g(x)$ is the inverse of $f(x) = x^3e^{x^2}$. Hint: $f(1) = e$.

(A) $\frac{1}{5e}$

(B) $\frac{5}{e}$

(C) $-\frac{5}{e}$

(D) $-\frac{1}{5e}$

(E) $\frac{1}{4e}$

10. Recall that $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$ and $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$. Find $\frac{dy}{dx}$, where $y = \arctan\left(\frac{x}{4}\right)$.

(A) $\frac{1}{4(1+x^2)}$

(B) $\frac{4}{16+x^2}$

(C) $\frac{4}{\sqrt{16-x^2}}$

(D) $\frac{4}{1+x^2}$

(E) $\frac{4}{\sqrt{1-x^2}}$

Free Response Questions: Show your work!

11. A road perpendicular to a highway leads to a farmhouse located 16 km from the highway. An automobile traveling on the highway passes through this intersection at a speed of 90 km/h.

How fast is the distance between the automobile and the farmhouse increasing when the automobile is 12 km past the intersection of the highway and the road? (Draw a picture and label the picture to represent the situation in this problem.)

Free Response Questions: Show your work!

12. (a) Assume that $f(x)$ is a differentiable function. The equation of the tangent line at $x = 7$ to the graph of $y = f(x)$ is $y = 3x - 10$. Find $f(7)$ and $f'(7)$.

- (b) Find the coordinates of the points on the graph of $y = x^3 + 6x^2 + 5x + 2$ where the tangent line is parallel to the line $y = -4x + 5$.

Free Response Questions: Show your work!

13. An object is dropped from a height of 640 meters. As usual, we ignore all air resistance in this problem. Use Galileo's formula $s(t) = s_0 + v_0t - \frac{1}{2}gt^2$, where $g = 9.8 \text{ m/s}^2$, to answer the following questions. Be sure to use correct units.

(a) When does the object hit the ground?

(b) What is the object's velocity when it hits the ground?

Free Response Questions: Show your work!

14. (a) State the definition of the derivative of a function $f(x)$.

(b) Compute the derivative of $f(x) = \frac{1}{x^2}$ using the definition of a derivative as a limit. (You will not receive any credit if you use a different method.)

Free Response Questions: Show your work!

15. An expanding sphere has radius $r = t^2 + t$ cm at time t in seconds, $t > 0$. In this problem you may use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere of radius r . Be sure to use correct units.

(a) Find the rate of change of the volume of the sphere with respect to time when the radius is 2 cm.

(b) Find the rate of change of the volume of the sphere with respect to time when $t = 3$.