Exam 2

Form A

Name: ________________________ Section and/or TA: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 12 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)   **7** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)   **8** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)   **9** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)   **10** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)   **11** (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)   **12** (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Suppose that $f(x) = \begin{cases} x+2 & \text{if } x \leq 2 \\ Ax+B & \text{if } 2 < x \leq 4 \\ 7x-12 & \text{if } x > 4 \end{cases}$

   Find the values of A and B which make $f(x)$ continuous everywhere.

   A. $A = 6,\ B = 8$

   B. $A = -6,\ B = 8$

   C. $A = 6,\ B = -8$

   D. $A = -6,\ B = -8$

   E. There is no solution.

2. Find the horizontal asymptotes of $f(x) = \dfrac{e^x}{1+e^x}$.

   A. $y = 0$

   B. $y = 1$

   C. $y = 1/2$

   D. $y = 0$ and $y = 1$

   E. $y = -1$

3. Which of the following statements is true if $f(x)$ is defined by

$$f(x) = \begin{cases} \sin x & \text{if } x \leq -\pi/4 \\ \cos x & \text{if } x > -\pi/4 \end{cases}$$

A. $f(x)$ is continuous at $x = -\pi/4$.
B. $f(x)$ has a jump discontinuity at $x = -\pi/4$.
C. $f(x)$ has an infinite discontinuity at $x = -\pi/4$.
D. $f(x)$ is not defined at $x = -\pi/4$.
E. There is not enough information to determine the continuity of $f(x)$ at $x = -\pi/4$.

4. Suppose that f is a continuous function on the interval $[0, 5]$ and we know that

$$f(0) = 1,\ f(1) = -1,\ f(2) = 1,\ f(3) = -1,\ f(4) = 1, \text{ and } f(5) = -1.$$

Which of the following statements are true for any such f?

A. There are at least five solutions of the equation $f(x) = 0$ in the interval $[0, 5]$.
B. There are at most five solutions of the equation $f(x) = 0$ in the interval $[0, 5]$.
C. There are exactly five solutions of the equation $f(x) = 0$ in the interval $[0, 5]$.
D. The equation $f(x) = -1$ has at most three solutions in the interval $[0, 5]$.
E. The equation $f(x) = 1$ has exactly three solutions in the interval $[0, 5]$.

5. Find a formula for $\dfrac{dy}{dx}$ in terms of x and y, where $x^2 + xy + y^2 = 1$.

   A. $\dfrac{dy}{dx} = -\dfrac{2x+y}{x+y}$

   B. $\dfrac{dy}{dx} = -\dfrac{2x}{x+2y}$

   C. $\dfrac{dy}{dx} = \dfrac{2x+y}{x+2y}$

   D. $\dfrac{dy}{dx} = -\dfrac{2x+y}{2y}$

   E. $\dfrac{dy}{dx} = -\dfrac{2x+y}{x+2y}$

6. Find all values of x where $f'''(x) = 0$ when $f(x) = xe^{2x}$.

   A. $x = 3/2$

   B. $x = 2/3$

   C. $x = -3/2$

   D. $x = -2/3$

   E. $x = -1$

7. Find $f'(x)$ in terms of $g'(x)$ where $f(x) = [g(x)]^4$.

   A. $f'(x) = 4[g(x)]^3$

   B. $f'(x) = 4[g(x)]^3 g'(x)$

   C. $f'(x) = 4[g'(x)]^3$

   D. $f'(x) = 4[gx]^3[xg' + g]$

   E. $f'(x) = 4g'(x)$

8. Find the derivative of $g(t) = \tan(\cos(2t))$.

   A. $g'(t) = 2\sin(2t)\sec^2(\cos(2t))$

   B. $g'(t) = \sin(2t)\sec^2(\cos(2t))$

   C. $g'(t) = -\sin(2t)\sec^2(\cos(2t))$

   D. $g'(t) = -2\sin(2t)\sec^2(\cos(2t))$

   E. $g'(t) = -2\sec^2(\sin(2t))$

9. Find the derivative of

$$g(x) = x^5 \ln(9x).$$

A. $g'(x) = x^4(1 + 5\ln(9x))$

B. $g'(x) = 1 + \dfrac{\ln(9x)}{9x}$

C. $g'(x) = x^4\left(\dfrac{1}{9} + 5\ln(9x)\right)$

D. $g'(x) = \dfrac{5}{9}x^3$

E. $g'(x) = x^4(5\ln(9x) - 1)$

10. Differentiate

$$f(x) = \frac{x^6}{1 - x^5}$$

A. $f'(x) = \dfrac{6x^5}{1 - 5x^4}$

B. $f'(x) = \dfrac{(x^5 - 1)^2}{x^5(6x - 5)}$

C. $f'(x) = \dfrac{x^5(6x - 5)}{(1 - x^5)^2}$

D. $f'(x) = \dfrac{x^5(1 - x^5)}{(6 - x^5)^2}$

E. $f'(x) = \dfrac{x^5(6 - x^5)}{(1 - x^5)^2}$

11. Suppose that $F(x) = f(g(x))$ and $g(14) = 2$, $g'(14) = 5$, $f'(14) = 15$, and $f'(2) = 11$. Find $F'(14)$

A. $F'(14) = 140$

B. $F'(14) = 55$

C. $F'(14) = 24$

D. $F'(14) = 20$

E. $F'(14) = 17$

12. If f and g are continuous functions with $f(9) = 6$ and $\lim_{x\to 9}[2f(x) - g(x)] = 9$, find $g(9)$.

A. $g(9) = 21$

B. $g(9) = 24$

C. $g(9) = 3$

D. $g(9) = 15$

E. $g(9) = 12$

Free Response Questions
Show all of your work

13. Find the derivatives of the following functions.

(a) $f(x) = \ln(\cos(2x))$.

(b) $g(x) = \dfrac{4}{x^5} - \dfrac{8}{x^4} - \dfrac{3}{x^3} + 700$.

(c) $h(x) = 4\ln(x^2 e^x)$.

14. (a) Find the equation of the tangent line to $2xy^2 - 5x^2y + 192 = 0$ at the point $(4, 4)$.

(b) Find $\lim_{x \to 0} \dfrac{\sin(3x)}{7x}$

15. Let $f(x) = \dfrac{x^2}{x+6}$.

(a) Find the derivative $f'(x)$.

(b) Find the equation of the tangent line to $f(x)$ at the point where $x = 4$.

16. This problem concerns the definition of the derivative using limits.

    (a) State the formal definition of the derivative of a function $f(x)$ at the point $x = a$.
    Hint: Your definition should involve a limit.

    (b) **Using the formal definition of derivative and the limit laws**, find the derivative of the function $f(x) = 2x^2 - 1$. An answer that is unsupported or uses differentiation rules will receive **no credit**.