Name: ______________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

This page is left blank.

Multiple Choice Questions

1. (5 points) Let $f(x) = xe^x$. Find $f'(1)$.

   A. $3e$

   B. 0

   C. e

   D. $2e$

   E. $4e$

Solution: We differentiate with the product rule, $f'(x) = xe^x + e^x = (x+1)e^x$. Setting $x = 1$, gives $f'(1) = 2e$.

2. (5 points) Find the derivative $g'(1)$ for $g(x) = \dfrac{1}{x^2+1}$.

   A. 2

   B. $1/2$

   C. -1

   D. 1

   E. $-1/2$

3. (5 points) Let $f(x) = \sqrt{4 - 2x}$. Find $f'(0)$.

A. $1/4$

B. 1

C. $1/2$

D. $-1/4$

E. $-1/2$

4. (5 points) Which of the expressions is equal to the limit $\lim_{h \to 0} \frac{\sin(h)}{h}$?

A. $-\cos'(0) = -\frac{d}{dx}\{\cos(x)\}\Big|_{x=0}$

B. $\sin'(0) = \frac{d}{dx}\{\sin(x)\}\Big|_{x=0}$

C. $\cos'(0) = \frac{d}{dx}\{\cos(x)\}\Big|_{x=0}$

D. $-\sin'(0) = -\frac{d}{dx}\{\sin(x)\}\Big|_{x=0}$

E. The limit does not exist

5. (5 points) Suppose $f(1) = 2$ and $f'(1) = 3$, $f'(0) = 5$ and $g(x) = f(\cos(2x))$ Find the derivative $g'(0)$.

   A. 0

   B. -6

   C. 6

   D. -10

   E. 10

6. (5 points) Suppose the position of an object at time t is $p(t) = \sin(2t)$. Select the time t where the velocity is positive and the acceleration is negative.

   A. $\pi/4$

   B. $\pi/8$

   C. 0

   D. π

   E. $\pi/2$

7. (5 points) Consider the triangle below. Give an expression for the angle u.

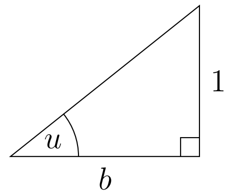
   A. $u = \text{arcsec}(b)$

   B. $u = \arctan(b)$

   C. $u = \arctan(1/b)$

   D. $u = \arcsin(b)$

   E. $u = \arccos(b)$



8. (5 points) Find the smallest, positive value t for which the function $f(t) = t + 2\cos(t)$ has a horizontal tangent line.

   A. $\pi/2$

   B. $\pi/3$

   C. $2\pi/3$

   D. $\pi/4$

   E. $\pi/6$

9. (5 points) Let $f(x) = \ln((x^2+1)(2x+5))$. Find the derivative $f'(x)$.

A. $\dfrac{2x(2x+5) - 2(x^2+1)}{(x^2+1)^2(2x+5)^2}$

B. $\dfrac{2x(2x+5) - 2(x^2+1)}{(x^2+1)^2(2x+5)}$

C. $\dfrac{2x(2x+5) + 2(x^2+1)}{(x^2+1)^2(2x+5)}$

D. $\dfrac{2x}{x^2+1} + \dfrac{2}{2x+5}$.

E. $\dfrac{2x(2x+5) + 2(x^2+1)}{(x^2+1)^2(2x+5)^2}$

Solution: We use the properties of logarithms to write $\ln((x^2+1)(2x+5)) = \ln(x^2+1) + \ln(2x+5)$. Differentiating

$$f'(x) = \frac{2x}{x^2+1} + \frac{2}{2x+5}.$$

10. (5 points) If $t = \sin(u)$ and $u \in (-\pi/2, \pi/2)$, find $\tan(u)$.

A. $-\sqrt{1-t^2}/t$

B. $t/\sqrt{1+t^2}$

C. $t/\sqrt{1-t^2}$

D. $-t/\sqrt{1-t^2}$

E. $\sqrt{1-t^2}/t$

11. (5 points) Consider the hyperbola defined by the equation $\frac{x^2}{3} - \frac{y^2}{2} = 1$. Find the slope of the tangent line to the hyperbola at the point $(-3, 2)$.

A. 1

B. -1

C. $9/4$

D. $2/3$

E. $-2/3$

12. (5 points) Let $f(x) = e^{2x}$. Find the 20^{th} derivative $f^{(20)}(0)$.

A. e^{20}

B. 1

C. 2^{20}

D. 0

E. 20

Free response questions, show all work

13. (10 points) Consider the curve defined by the equation

$$(x^2+1)y^3 - 2x = 14x^2.$$

(a) Find the derivative $\frac{dy}{dx}$ along the curve.

(b) Let $x_0 = 1$ and find the point $(1, y_0)$ which lies on the curve.

(c) Find the equation of the tangent line to the curve at $x_0 = 1$.

Solution: a) Differentiating we have

$$2xy^3 + (x^2+1)\cdot 3y^2\frac{dy}{dx} - 2 = 28x$$

Solving for the derivative gives

$$dydx = \frac{28x + 2 - 2xy^3}{3y^2(x^2+1)}.$$

b) We substitute 1 for x and solve for y,

$$2y^3 - 2 = 14, \quad y^3 = 16/2 = 8, y = 2.$$

The point $(1, 2)$ lies on the curve.

c) The slope dy/dx is

$$\frac{dy}{dx} = \frac{28 + 2 - 2\cdot 8}{24} = \frac{7}{12}.$$

The tangent line passes through $(1, 2)$ with slope $7/12$ and has equation

$$(y-2) = \frac{7}{12}(x-1) \quad \text{or} \quad y = \frac{7}{12}x + \frac{17}{12}.$$

14. (10 points)

(a) State the mean value theorem.

(b) Suppose f is differentiable on $(-\infty, \infty)$, $f(2) = 3$ and $f'(x) \leq 7$ for x in the interval $(-\infty, \infty)$. Use the mean value theorem to find a number M so that $f(6) \leq M$.

Solution: a) *Theorem.* If f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b), then there is point c in (a, b) so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

b) Since f is differentiable on the $(-\infty, \infty)$ it is also continuous there and we may apply the mean value theorem on any interval.

Using the mean value theorem on the interval $[2, 6]$, we have a number c so that

$$\frac{f(6) - f(2)}{4} = f'(c) \leq 7.$$

Since $f(2) = 3$, the give $f(6) \leq f(2) + 28 = 31$.

a) Continuity, differentiable, intervals (3 points). Existence of c in (a, b) (accept $[a, b]$) (1 point), equation $f'(c) = (f(b) - f(a))/(b - a)$ (1 point).

b) 1 point, verify continuity and differentiability hypotheses, Obtain inequality $(f(6) - f(3))/4 \leq 7$ (2 points), solve to find $f(6) \leq 31$ (2 points).

15. (10 points) Suppose that a ball is thrown into the air at time $t = 0$ seconds so that its height above the ground after t seconds is $h(t) = -5t^2 + 30t$ meters.

    (a) Find the velocity at the instant the ball is thrown.

    (b) Find the time when the ball's velocity is zero.

    (c) Find the velocity of the ball as it hits the ground.

Solution: a) $v(t) = h'(t) = -10t + 30$. $v(0) = 30$ m/s.

b) Solve $v(t) = 0$. $-10t + 30 = 0$ if $t = 3$ seconds

c) The ball hits the ground when $h(t) = 0$ or $-5t^2 + 30t = 0$. Solving $t = 0$ or $t = 6$ seconds. At $t = 6$, the velocity is $h'(6) = -60 + 30 = -30$ meters/second.

a) Find $v(t)$ (2 points). Value of $v(0) = 30$ 1 point.

b) Equation $h'(t) = 0$, 1 point. Solution $t = 3$ seconds (1 point)

c) Attempt to solve $h(t) = 0$ (1 point), find $t = 6$ as solution 1 point. Find velocity $v(6)$ (2 points)

Units (1 point) if majority of answers have units.

Students may read $v(0) = 30$ directly from $h(t)$ in part a). Award two points for $h'(t)$ if they find h' somewhere.

16. (10 points) Let $f(x) = 2/x$.

(a) Find the slope of the tangent line to the graph of f at the point $(a, 2/a)$.

(b) Find the equation of the tangent line to the graph of f at the point $(a, 2/a)$.

(c) Find a so that the tangent line to the graph of f at $(a, 2/a)$ passes through the point $(0, 6)$.

Solution: a) The derivative of $f(x) = 2x^{-1}$ is $f'(x) = -2x^{-2}$. The slope at $x = a$ is $-2/a^2$.

b) The tangent line at $(a, 2/a)$ is $y - 2/a = (-2/a^2)(x - a)$ or $y = -\frac{2}{a^2}x + \frac{4}{a}$.

c) If the line passes through $(0, 6)$, we have $6 = -\frac{2}{a^2} \cdot 0 + \frac{4}{a}$ or $a = 4/6 = 2/3$.