Name: ______________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **11** (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E) **12** (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Let $f(x) = x\cos(2x)$. Find $f'(\pi/2)$.

   A. -1 B. π C. $-\pi$ D. 1 E. $1-\pi$

2. (5 points) If $f(x) = xe^{2x}$, find $f''(x)$.

   A. $(4x+3)e^{2x}$

   B. $(4x+4)e^{2x}$

   C. $(4x+2)e^{2x}$

   D. $4xe^{2x}$

   E. $(4x+1)e^{2x}$

3. (5 points) Suppose f is differentiable at 2 with $f(2) = 2$ and $f'(2) = 3$. Let $g(x) = \frac{f(x)}{x^2}$ and find $g'(2)$.

   A. 1/8 B. 1/2 C. 3/8 D. 0 E. 1/4

4. (5 points) Find the tangent line to $f(x) = x\ln(x)$ at $x = 1$.

   A. $y = x - 1$

   B. $y = x + 1$

   C. $y = x$

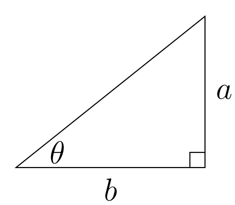
   D. $y = -x + 1$

   E. $y = -x - 1$

5. (5 points) Assume that x and y are related by the equation $y^2 + ax = 3a^2$ where $a \neq 0$ is a nonzero constant. Find dy/dx at the point $(2a, a)$.

A. $1/2$ B. $-1/2$ C. $-a/2$ D. $a/2$ E. 1

6. (5 points) In the right triangle at right, express b in terms of a and the angle θ.



A. $b = a\cos(\theta)$

B. $b = a\tan(\theta)$

C. $b = a/\tan(\theta)$

D. $b = a\sec(\theta)$

E. $b = a\sin(\theta)$

7. (5 points) Suppose $\cos(\theta) = t$ and $0 \leq \theta \leq \pi$. Express $\sin(\theta)$ without using trigonometric or inverse trigonometric functions.

   A. $\sin(\theta) = -\sqrt{1-t^2}$

   B. $\sin(\theta) = \sqrt{1-t^2}$

   C. $\sin(\theta) = -\sqrt{1+t^2}$

   D. $\sin(\theta) = -t/\sqrt{t^2-1}$

   E. $\sin(\theta) = \sqrt{1+t^2}$

8. (5 points) Let $f(x) = \arctan(3x)$. Find $f'(x)$.

   A. $\dfrac{1}{1+x^2}$

   B. $\dfrac{3}{1+x^2}$

   C. $\dfrac{1}{1+9x^2}$

   D. $\dfrac{3}{1+9x^2}$

   E. $\dfrac{1}{1+3x^2}$

9. (5 points) Assume the acceleration of gravity is meters/second^2 in the downward direction. A ball is thrown upward from a position 100 meters above the ground and with a speed of 30 meters/second. Find the height of the ball above the ground 3 seconds after it is thrown.

   A. 235 meters

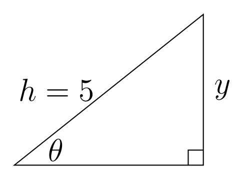
   B. 100 meters

   C. 145 meters

   D. 190 meters

   E. 55 meters

10. (5 points) In the triangle pictured at right, $h = 5$ is fixed and y is varying. Find the derivative $d\theta/dy$ when $y = 4$.



A. 1/4 B. 1/5 C. 3/4 D. 4/3 E. 1/3

11. (5 points) Let $f(x) = \cos(x)$ and find $f^{(101)}(x)$, the 101st derivative of f.

A. $\sin(x)$

B. 0

C. $-\sin(x)$

D. $-\cos(x)$

E. $\cos(x)$

12. (5 points) Find $Q(x)$, the quadratic approximation to $f(x) = e^{2x}$ at $x = 0$.

A. $Q(x) = 1 + 2x + 4x^2$

B. $Q(x) = 1 + x + x^2$

C. $Q(x) = 1 + 2x + x^2$

D. $Q(x) = 1 + 2x + 2x^2$

E. $Q(x) = 1 + x + 2x^2$

Free response questions, show all work

13. (10 points) (a) Find the equation of the tangent line to $y = x^2$ at the point (a, a^2).

    (b) Use your answer to part a) to find an equation that a must satisfy if the tangent line to $y = x^2$ at (a, a^2) passes through the point $(2, 0)$.

    (c) Solve the equation from part b) and find all tangent lines to the graph of $y = x^2$ which pass through the point $(2, 0)$.

14. (10 points) (a) Find all values b so that the point $(2, b)$ lies on the ellipse $x^2 + 3y^2 = 7$.

(b) Suppose that $x^2 + 3y^2 = 7$. Find $\frac{dy}{dx}$ in terms of x and y.

(c) For each of the points $(2, b)$ on the ellipse that your found in part a), find the equation of the line that is tangent to the ellipse at $(2, b)$. Simplify to write the equation of each tangent line in slope-intercept form.

15. (10 points) (a) State the mean value theorem.

(b) Let $f(x) = \sin(2x)$. Find the largest possible value of $f'(x)$ for x in $(-\infty, \infty)$.

(c) Use the mean value theorem on the interval $[0, 1/6]$ and your answer to part b) to find a value M so that $f(1/6) \leq M$.

16. (10 points) (a) Find the linear and quadratic approximations to $f(x) = \sqrt{x}$ at 25.

(b) Use the linear approximation to find an approximate value for $\sqrt{22}$. Your answer should be correctly rounded to three decimal places or more accurate.

(c) Use the quadratic approximation to find an approximate value for $\sqrt{29}$. Your answer should be correctly rounded to three decimal places or more accurate.

Hint: In parts b) and c) you may check if your answer is reasonable by comparing it with the values of $\sqrt{22}$ and $\sqrt{29}$ from a calculator. However, you will need to show that you found your approximations using the linear and quadratic approximations to receive full credit for this problem.