MA 113 Calculus I — Spring 2017

Exam 2 — Tuesday, March 7, 2017

Name: ______________________________

Section: ______________________________

Last 4 digits of student ID #: ________

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

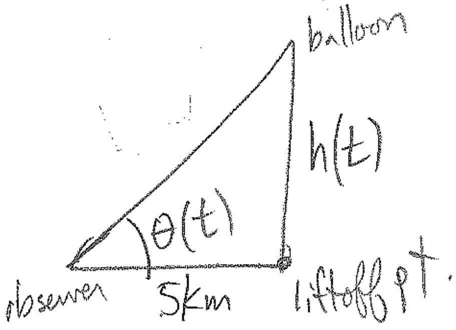
Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Free Response Questions: Show your work!

16. A hot air balloon rising vertically is tracked by an observer located 5 km from the liftoff point.

    (a) Find an equation to relate the height of the balloon and the angle of the observer's line-of-sight.



$$\tan \theta(t) = \frac{h(t)}{5}. \quad (*)$$

    (b) At a certain moment, the angle between the observer's line-of-sight and the horizontal is $\frac{\pi}{3}$, and it is changing at a rate of 0.1 rad/min. How fast is the balloon rising?

Apply $\frac{d}{dt}$ to $(*)$, get $\sec^2\theta(t) \cdot \frac{d\theta}{dt} = \frac{dh}{dt} \cdot \frac{1}{5}$.

$$\Rightarrow \sec^2\left(\frac{\pi}{3}\right) \cdot 0.1 = \frac{dh}{dt} \cdot \frac{1}{5}$$

$$\Rightarrow \frac{dh}{dt} = 0.5 \cdot \sec^2\left(\frac{\pi}{3}\right) \text{ km/min}.$$

Free Response Questions: Show your work!

17. (a) Suppose that $x^6 + y^6 = 2$. Find the slope of the tangent line to the curve defined by this equation at the point $(1, -1)$.

$$6x^5 + 6y^5 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x^5}{y^5}.$$

$$\Rightarrow \text{slope is } -\frac{(1)^5}{(-1)^5} = 1.$$

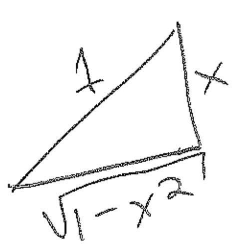
(b) Use implicit differentiation to find the derivative of $\arcsin(x)$. Note: writing only the formula for the derivative of $\arcsin(x)$ will receive no credit, you must show how you obtain this formula.

Let $g(x) = \arcsin(x)$. Then

$$\sin(g(x)) = x$$

$$\Rightarrow \cos(g(x)) \cdot g'(x) = 1$$

$$\Rightarrow g'(x) = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}}.$$



Free Response Questions: Show your work!

18. Let $f(x) = e^{\cos(x/2)}$.

(a) Find the derivative $f'(x)$.

$$f'(x) = e^{\cos\left(\frac{x}{2}\right)} \cdot \left(-\sin\left(\frac{x}{2}\right)\right) \cdot \frac{1}{2}.$$

(b) Find the equation to the slope of the tangent line to $f(x)$ at the point where $x = -\pi$.

$$f'(-\pi) = e^{\cos\left(\frac{-\pi}{2}\right)} \cdot \left(-\sin\left(\frac{-\pi}{2}\right)\right) \cdot \frac{1}{2}$$

So, $y - e^{\cos\left(\frac{\pi}{2}\right)} = e^{\cos\left(\frac{-\pi}{2}\right)} \cdot \left(-\sin\left(\frac{-\pi}{2}\right)\right) \cdot \frac{1}{2}\left(x+\pi\right).$

is the answer.

Free Response Questions: Show your work!

19. This problem concerns the definition of the derivative using limits.

(a) State the formal definition of the derivative of a function $f(x)$ at the point $x = a$. *Hint*: Your definition should involve a limit.

See textbook.

(b) **Using the formal definition of derivative and the limit laws**, find the derivative of the function $f(x) = \dfrac{1}{3x}$. An answer that is unsupported or uses differentiation rules will receive **no credit**.

$$\lim_{h\to 0} \frac{\frac{1}{3(x+h)} - \frac{1}{3x}}{h} = \lim_{h\to 0} \frac{3x - 3(x+h)}{h \cdot 3x \cdot 3(x+h)} = \lim_{h\to 0} \frac{-h}{3h \cdot x \cdot (x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{3x(x+h)} = \frac{-1}{3x^2}.$$