Name: ________________________________

Section: ______________________________

Last 4 digits of student ID #: ________

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your final answers in the front page answer box on the front page of your exam.

2. Carefully check your answers. No credit will be given for answers other than those indicated on the front page answer box.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),

2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

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**True/False**

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**Multiple Choice**

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**Overall Exam Scores**

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Record the correct answer to the following problems on the front page of this exam.

1. True or False: If $f$ and $g$ are differentiable, then the derivative of $fg$ is $f'g'$.

2. True or False: If $f'(x) = -\sin(x)$, then $f(x)$ must be equal to $\cos(x)$.

3. True or False: If $f(x)$ is a polynomial, then the derivative of $f(x)$ is also a polynomial.

4. True or False: The derivative of $\ln(2x)$ is $\frac{1}{2x}$.

5. True or False: The derivative of $e^x$ is $e^x$.

6. Find a function $f$ and a number $a$ so that the following limit represents a derivative $f'(a)$.

$$\lim_{h \to 0} \frac{e^{(1+h)^2} - e}{h}$$

(A) $f(x) = e^{x^2}$, $a = 1$
(B) $f(x) = e^{2x}$, $a = 1$
(C) $f(x) = e^{(1+x)^2}$, $a = 1$
(D) $f(x) = e^{2x}$, $a = 0$
(E) None of the above

7. Find the second derivative of $f(x) = 3^{30}$.

(A) $30 \cdot 29 \cdot 3^{28}$
(B) 0
(C) $\ln(3)3^{30}$
(D) 3
(E) None of the above
8. Suppose that $h(t) = \ln (5t^3 + 2t^2 + t + 2)$. Then $h'(0)$ is:

(A) 0
(B) 1
(C) $1/\ln(2)$
(D) $1/2$
(E) None of the above

9. Find a formula for $\frac{dy}{dx}$ in terms of $x$ and $y$, where $x^2 + xy + y^2 = 1$.

(A) $-(2x + y)/(x + y)$
(B) $-2x/(x + 2y)$
(C) $(2x + y)/(x + 2y)$
(D) $-(2x + y)/(x + 2y)$
(E) None of the above.

10. For three differentiable functions $f$, $g$, and $h$, the derivative of $f \cdot g \cdot h$ is:

(A) $f'gh'$
(B) $fg'h + fgh'$
(C) $f'gh + fg'h + fgh'$
(D) $f'gh + fg'h$
(E) None of the above
Record the correct answer to the following problems on the front page of this exam.

11. A 5 meter ladder leans against a wall. The bottom of the ladder is 3 meters from the wall at $t = 0$ seconds and slides away from the wall at a rate of 0.3 meters per second. Find the velocity of the top of the ladder at time $t = 0$ seconds.

(A) $-9/4 \text{ m/s}$  
(B) $-3/4 \text{ m/s}$  
(C) $9/40 \text{ m/s}$  
(D) $-9/40 \text{ m/s}$  
(E) None of the above

12. Suppose that $f(x) = \sin x + \cos 2x$. Find $f^{(8)}(x)$

(A) $\sin x + 128 \cos x$  
(B) $\sin x + 256 \cos x$  
(C) $-\sin x - 128 \cos x$  
(D) $-\cos x - 256 \sin x$  
(E) None of the above

13. Find the derivative of $f(x) = \pi^{2x}$.

(A) $(2x - 1)\pi^{2x-1}$  
(B) $2 \cdot \pi^{2x}$  
(C) $(2\pi)\pi^{2x}$  
(D) $\ln(\pi)\pi^{2x}$  
(E) None of the above
14. Find all values of $x$ where $f^{(3)}(x) = 0$ when $f(x) = xe^{2x}$

(A) $-3/2$
(B) $-2/3$
(C) $3/2$
(D) $-1$
(E) None of the above

15. If $f(x)$ is invertible, $g(x) = f^{-1}(x)$, and we have $f(3) = 7$ and $f'(3) = \sqrt{2}$, find the slope of the tangent line to $g(x)$ at the point $(7, 3)$.

(A) $3\sqrt{2}$
(B) $3/7$
(C) $1/\sqrt{2}$
(D) $\sqrt{2}$
(E) $1/3$
16. A hot air balloon rising vertically is tracked by an observer located 5 km from the liftoff point.

(a) Find an equation to relate the height of the balloon and the angle of the observer’s line-of-sight.

(b) At a certain moment, the angle between the observer’s line-of-sight and the horizontal is \( \frac{\pi}{3} \), and it is changing at a rate of 0.1 rad/min. How fast is the balloon rising?
17. (a) Suppose that $x^6 + y^6 = 1$. Find the slope of the tangent line to the curve defined by this equation at the point $(1, -1)$.

(b) Use implicit differentiation to find the derivative of $\arcsin(x)$. Note: writing only the formula for the derivative of $\arcsin(x)$ will receive no credit, you must show how you obtain this formula.
18. Let \( f(x) = e^{\cos(x/2)} \).

(a) Find the derivative \( f'(x) \).

(b) Find the equation to the slope of the tangent line to \( f(x) \) at the point where \( x = -\pi \).
19. This problem concerns the definition of the derivative using limits.

(a) State the formal definition of the derivative of a function $f(x)$ at the point $x = a$.  
*Hint:* Your definition should involve a limit.

(b) **Using the formal definition of derivative and the limit laws,** find the derivative of the function $f(x) = \frac{1}{3x}$. An answer that is unsupported or uses differentiation rules will receive **no credit**.