Exam 2

Form A

Multiple Choice Questions

1. Find the derivative of $\sin(x^3 + x)$.

   A. $\cos(x^3 + x)$

   B. $\sin(3x^2 + 1)$

   C. $\cos(3x^2 + 1)$

   D. $(3x^2 + 1)\cos(x^3 + x)$

   E. $(3x^2 + 1)\sin(x^3 + x)$

2. Suppose $f(3) = 5$ and $f'(3) = -3$ and let $g(x) = 2xf(x)$. Find $g'(3)$

   A. -18

   B. -15

   C. -8

   D. -6

   E. 28

3. Find $f(3)$ and $f'(3)$, assuming that the tangent line to $y = f(x)$ at $x = 3$ has equation $y = 3x - 2$.

   A. $f(3) = 3,\ f'(3) = -2$

   B. $f(3) = -2,\ f'(3) = 3$

   C. $f(3) = 2,\ f'(3) = 3$

   D. $f(3) = 7,\ f'(3) = 3$

   E. $f(3) = 11,\ f'(3) = 3$

4. Find a formula for $\dfrac{dy}{dx}$ in terms of x and y where $x^2y - xy^2 = 2$.

   A. $\dfrac{dy}{dx} = \dfrac{y^2 - 2xy}{x^2 - 2}$

   B. $\dfrac{dy}{dx} = -\dfrac{y^2 + 2xy}{x^2 + 2xy}$

   C. $\dfrac{dy}{dx} = \dfrac{y^2 - 2x}{x^2 - 2y}$

   D. $\dfrac{dy}{dx} = \dfrac{x^2}{2xy}$

   E. $\dfrac{dy}{dx} = \dfrac{y^2 - 2xy}{x^2 - 2xy}$

5. Find the slope of the tangent line to the graph of $f(x) = x^3 \ln(x^2)$ at $x = e$.

   A. $8e^2$

   B. $6e^2 + 2e$

   C. $6e^2 + e$

   D. $6e$

   E. 3

6. Find an equation of the tangent line to $4x^2 + 9y^2 = 72$ at the point $(3, -2)$.

   A. $y = -\frac{2}{3}x$

   B. $y = \frac{3}{2}x - \frac{13}{2}$

   C. $y = \frac{2}{3}x + \frac{13}{3}$

   D. $y = \frac{2}{3}x - 4$

   E. $y = \frac{2}{3}x$

7. Let $h(x) = \dfrac{x^3+1}{x^2+1}$. Find $h'(1)$.

   A. -2

   B. $1/2$

   C. 1

   D. $3/2$

   E. 2

8. A cantaloupe is dropped off a tall building so that its height in meters at time t in seconds is $h(t) = -4.9t^2 + 98$. Find the velocity when it hits the ground. Give your answer correctly rounded to one decimal place.

   A. -43.4

   B. -43.6

   C. -43.8

   D. -44.2

   E. -44.4

9. Find the derivative of $g(x) = \tan(3x) + \sin(x^2)$.

   A. $g'(x) = \sec^2(3x) + \cos(x^2)$

   B. $g'(x) = 3\sec^2(3x) + 2x\cos(x^2)$

   C. $g'(x) = \sec^2(3x) + \cos(2x)$

   D. $g'(x) = 3\tan(3x) + 2x\cos(x^2)$

   E. $g'(x) = 3\tan(3x) + \cos(2x)$

10. Chromium-51 has a half-life of 28 days. A sample has a mass of 50 mg initially. Find the mass remaining after 30 days rounded to two decimal places.

    A. 8.70 mg

    B. 22.75 mg

    C. 23.79 mg

    D. 24.81 mg

    E. 25.00 mg

11. Find $f'(x)$ in terms of $g(x)$ and $g'(x)$ where $f(x) = [g(x)]^3$.

A. $f'(x) = 3g'(x)$

B. $f'(x) = 3[g(x)]^2$

C. $f'(x) = 3[g'(x)]^2$

D. $f'(x) = 3[g(x)]^2 g'(x)$

E. $f'(x) = 3[g(x)]^2(xg'(x) + g(x))$

12. Differentiate

$$f(x) = \frac{x^5}{1 - x^4}.$$

A. $f'(x) = \dfrac{5x^4}{1 - 4x^3}$

B. $f'(x) = \dfrac{(1 - x^4)^2}{x^4(5x - 4)}$

C. $f'(x) = \dfrac{x^4(5x - 4)}{(1 - x^4)^2}$

D. $f'(x) = \dfrac{x^4(1 - x^4)}{(5 - x^4)^2}$

E. $f'(x) = \dfrac{x^4(5 - x^4)}{(1 - x^4)^2}$

13. Find the derivative of

$$g(x) = x^5 \ln(9x).$$

A. $g'(x) = x^4(1 + 5\ln(9x))$

B. $g'(x) = x^4\left(\frac{1}{9} + 5\ln(9x)\right)$

C. $g'(x) = 1 + \frac{\ln(9x)}{9x}$

D. $g'(x) = \frac{5}{9}x^3$

E. $g'(x) = x^4(5\ln(9x) - 1)$

14. Let $f(x) = e^{(x^2)}$. Find $f''(x)$.

A. $(2 + 2x^2)e^{(x^2)}$

B. $(2 + 4x^2)e^{(x^2)}$

C. $(2 + 2x)e^{(x^2)}$

D. $(2 + 4x)e^{(x^2)}$

E. $2xe^{(x^2)}$

Free Response Questions
Show all of your work

15. (a) Find dy/dx for the curve $y^2 - 5xy + 6x^2 = 2$.

Solution:

$$\begin{aligned} \frac{d}{dx}(y^2 - 5xy + 6x^2) &= \frac{d}{dx}(2) \\ 2y\frac{dy}{dx} - (5y + 5x\frac{dy}{dx}) + 12x &= 0 \\ (2y - 5x)\frac{dy}{dx} &= 5y - 12x \\ \frac{dy}{dx} &= \frac{5y - 12x}{2y - 5x} \end{aligned}$$

(b) Find the slope of the tangent line to $y^2 - 5xy + 6x^2 = 2$ at the point $(1, 4)$.

Solution: At $(1, 4)$ the slope is $m = \left.\frac{dy}{dx}\right|_{(1,4)} = \frac{5 \cdot 4 - 12 \cdot 1}{2 \cdot 4 - 5 \cdot 1} = \frac{8}{3}$.

(c) Find an equation of the tangent line to $y^2 - 5xy + 6x^2 = 2$ at the point $(1, 4)$.

Solution: The equation of the tangent line with slope $\frac{8}{3}$ passing through the point $(1, 4)$ is $y = 4 + \frac{8}{3}(x - 1)$.

16. Find the derivatives of the following functions

(a) $f(x) = \ln(\sec(x))$

Solution:

$$f'(x) = \frac{1}{\sec(x)} \cdot \sec(x)\tan(x) = \tan(x)$$

(b) $g(x) = x^2 e^{3x}$

Solution:

$$g'(x) = 2xe^{3x} + 3x^2 e^{3x}$$

(c) $h(x) = \dfrac{\sin(x)}{x^4}$

Solution:

$$h'(x) = \frac{x^4\cos(x) - 4x^3\sin(x)}{x^8} = \frac{x\cos(x) - 4\sin(x)}{x^5}$$

(d) $j(x) = \arctan(x^2)$

Solution:

$$j'(x) = \frac{1}{1+(x^2)^2}\cdot 2x = \frac{2x}{1+x^4}$$

(e) $k(x) = \dfrac{2}{x^2} - \dfrac{3}{x} + 5 + 4x^2 + x^9$

Solution:

$$k'(x) = -4x^{-3} + 3x^{-2} + 8x + 9x^8$$

17. The length of a rectangle is increasing at a rate of 12 cm/sec and its width is increasing at a rate of 4 cm/sec.

(a) Find an equation that relates the area (A) of the rectangle to its length (L) and its width (W).

Solution: $A = L \times W$.

(b) Find an equation that relates the rate of change of the area of the rectangle, dA/dt, to the rates of change of the length and the width, dL/dt and dW/dt.

Solution:

$$\frac{dA}{dt} = L\frac{dW}{dt} + W\frac{dL}{dt}$$

(c) If the length is 25 cm and the width is 15 cm, how fast is the area of the rectangle increasing?

Solution: We are given that $\dfrac{dL}{dt} = 12$ cm/sec and $\dfrac{dW}{dt} = 4$ cm/sec. We are also told that $L = 25$ cm and $W = 15$ cm, so

$$\frac{dA}{dt} = 25\cdot 4 + 15\cdot 12 = 280 \text{ cm}^2/\text{sec}$$