

Calculus I
Exam 3

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13 November 2001

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*). Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question. The total on this test is 100 points.

Name _____

Section _____

Question	Score	Total
1		10
2		5
3		5
4		5
5		10
6		5
7		6
8		10
9		16
10		10
11		10
12		8
Total		100

3. State the mean value theorem.

4. Prove that $\sqrt{9+x} \leq 3 + x/6$ for $x > 0$.

5. The next page gives the graph of a function f , list which of the x -coordinates $A - F$ are: (Be careful, each part may have more than one answer. Each point may appear zero, one or more times in your answers.)

(a) absolute maxima. _____

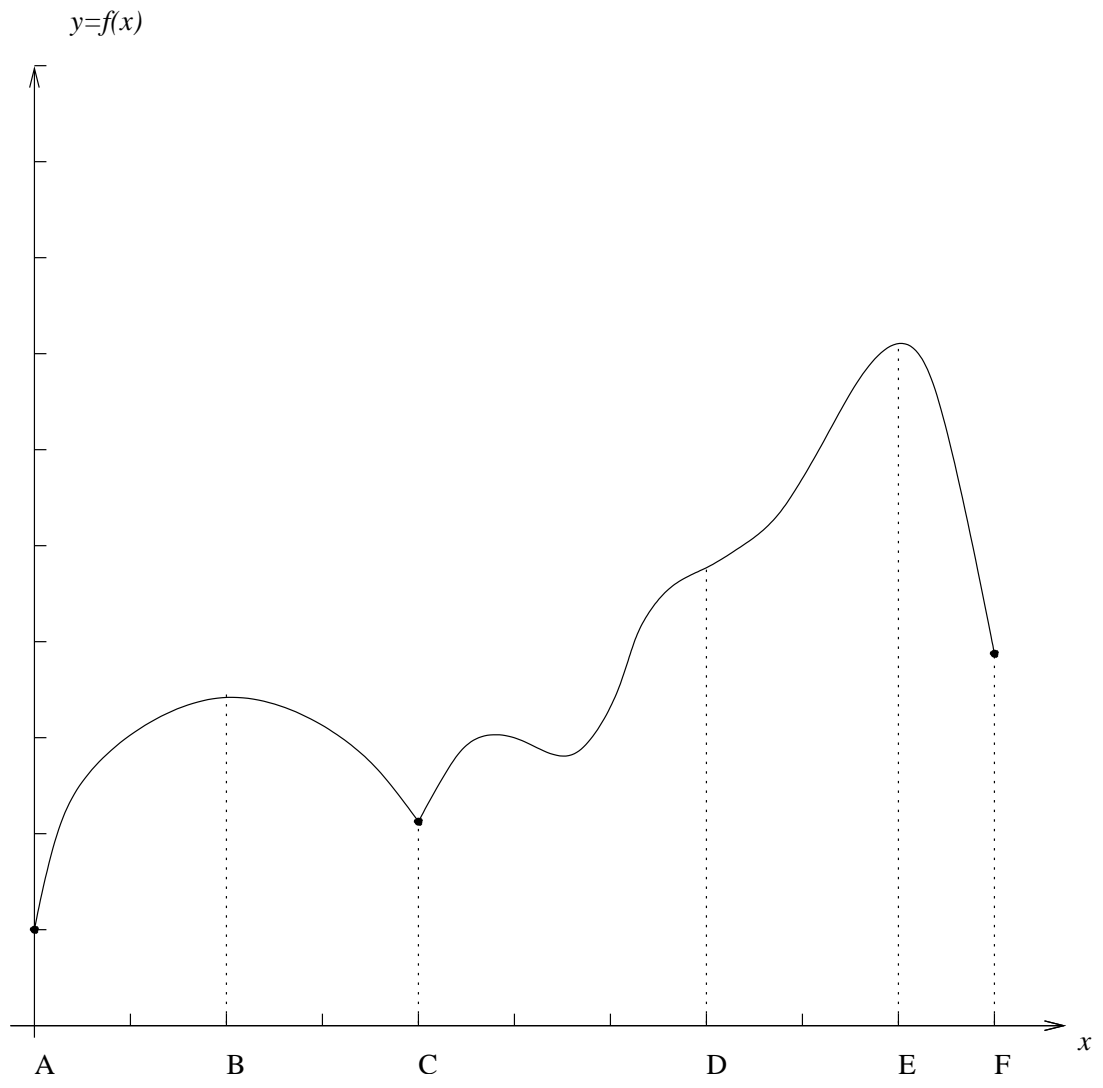
(b) absolute minima. _____

(c) critical numbers. _____

(d) local maxima. _____

(e) local minima. _____

(f) the x -coordinate of an inflection point. _____



6. If

$$f'(x) = \frac{x+1}{x^2+1} \quad \text{and} \quad f''(x) = \frac{-x^2-2x+1}{(x^2+1)^2}$$

Find all local extrema of $f(x)$. Determine if each local extrema is a local maximum or local minimum.

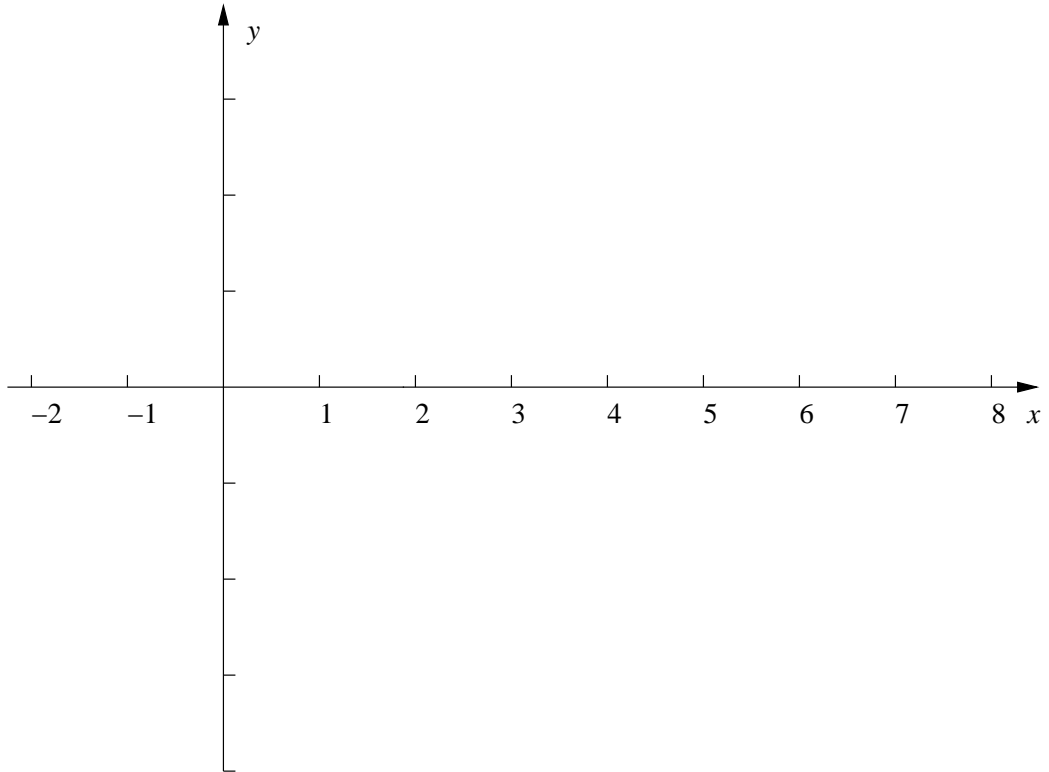
7. Find the limits.

(a) $\lim_{x \rightarrow \infty} \frac{x^2+1}{(1-x)(1+2x)}$

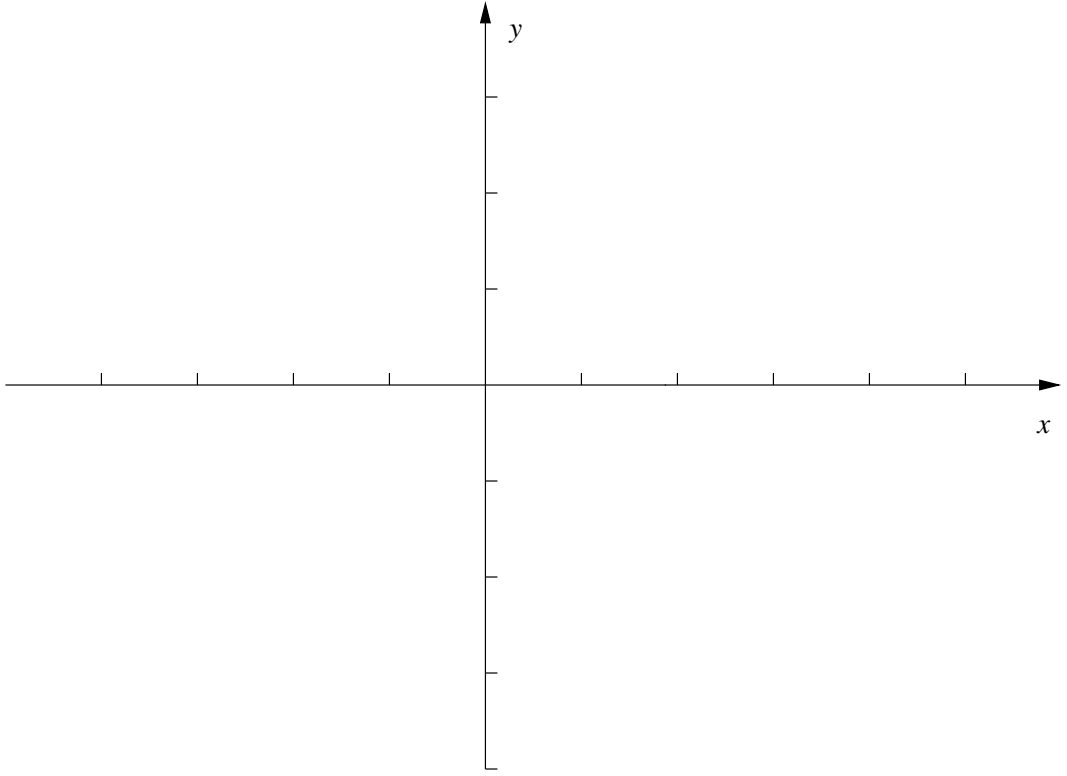
(b) $\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x+2})$.

8. Sketch the graph of a function $f(x)$ which satisfies the following information

Interval	$(-2, 0)$	$(0, 2)$	$(2, 4)$	$(4, 6)$
Sign of f'	-	-	+	-
Sign of f''	-	+	+	-



9. Let $f(x) = x^2 - x^3$. Use calculus to find the following information.
- (a) Intervals of increase and decrease for f .
 - (b) Intervals where the graph of f is concave up and concave down.
 - (c) Local maxima and minima for f .
 - (d) Inflection points.
 - (e) On the axes provided on the next page, draw a beautiful sketch of the graph of f which reflects the above information. Label the local extrema and inflection points.



10. Suppose a rectangle has one side on the x -axis and its other two vertices above the x -axis on the curve $y = 80 - x^4$. Find the dimensions of the rectangle satisfying these conditions and of largest possible area. Be sure to explain how you know you have found the absolute extreme value.

11. (a) Find a function which has $f'(x) = \cos x$ and $f(0) = 1$.
- (b) Find a function which has $f''(x) = 2x + 1$ and has $f(0) = 2$ and $f'(1) = 3$.

12. (a) Compute $\sum_{k=1}^5 (2k - 1)$.
(b) Compute $\sum_{k=2}^4 \cos(\pi k)$.