MA 113 — Calculus I Fall 2007
Exam 3 November 13, 2007

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. check answers when possible,
2. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
3. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.
You are to answer two of the last three questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		9
2		9
3		8
4		8
5		9
6		8
7		9
8		9
9		14
10		14
11		14
Free	3	3
		100

(1) Find the absolute maximum and absolute minimum value of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-3, 3]$.

Absolute maximum value: __________ Absolute minimum value: __________

(2) Consider the function $f(x) = x^4 - 4x^3 - 18x^2 + 5x + 1$. Use the concavity test to determine

(a) the interval(s) where the graph of f is concave upward,

(b) the interval(s) where the graph of f is concave downward,

(c) all inflection points of f.

If there are none, write NONE.

(a) Interval(s) where f is concave upward ____________________

(b) Interval(s) where f is concave downward ____________________

(c) Point(s) of inflection of f ____________________

(3) Let f be a function with **derivative** given by $f'(x) = 3(x-2)^2(x-1)^3(x+3)$. Use this derivative to determine

(a) the interval(s) where f is increasing,

(b) the interval(s) where f is decreasing,

(c) all the values of x where f has a local maximum,

(d) all the values of x where f has a local minimum.

If there are none, write NONE.

(a) Interval(s) where f increasing ____________________

(b) Interval(s) where f decreasing ____________________

(c) Value(s) where f has local maximum ____________________

(d) Value(s) where f has local minimum ____________________

(4) Use the limit laws to find the following limits:

(a) $\lim\limits_{x\to\infty} \dfrac{\sqrt{9x^8 + x}}{2x^4 - 6}$

(b) $\lim\limits_{x\to\infty} (x - \sqrt{x^2 + 5})$

(a) ____________ (b) ____________

(5) Find the most general antiderivative for each of the following functions:

(a) $f(x) = x^5 - 16x^3 + x - 2$

(b) $g(x) = \sin(x) + \cos(x)$

(c) $h(x) = 3\sqrt{x} - \dfrac{1}{x\sqrt{x}}$

(a) General antiderivative of f: ______________________

(b) General antiderivative of g: ______________________

(c) General antiderivative of h: ______________________

(6) A car is traveling along a straight road at 60 ft/s when the driver slams on the brakes in such a way that the acceleration is constant until the car stops. It took the car 5 seconds to stop.

(a) Determine the acceleration while braking.

(b) How far did the car travel after the brakes were applied?

(a) Acceleration ______________

(b) Distance the car traveled ______________

(7) It is known that a certain function f satisfies $\int_2^5 f(x)\,dx = 7$ and $\int_1^5 f(x)\,dx = 3$. Find the value of:

(a) $\int_1^5 [5 + 3f(x)]\,dx$

(b) $\int_1^2 f(x)\,dx$.

(a) $\int_1^5 [5 + 3f(x)]\,dx =$ ______________

(b) $\int_1^2 f(x)\,dx =$ ______________

(8) Interpret each the following integrals as an area. Graph the area and use geometry to find the value of each integral.

(a) $\displaystyle\int_1^4 \frac{1}{2} t \, dt$

(b) $\displaystyle\int_0^3 \sqrt{9 - t^2} \, dt.$

(a) $\displaystyle\int_1^4 \frac{1}{2} t \, dt =$ ______________

(b) $\displaystyle\int_0^3 \sqrt{9 - t^2} \, dt =$ ______________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(9) Let $g(x)$ be a function with domain $\{x \mid x \neq 2\}$ that is twice differentiable and satisfies the following conditions:

i) $\lim_{x\to\infty} g(x) = 1, \qquad \lim_{x\to-\infty} g(x) = 0,$

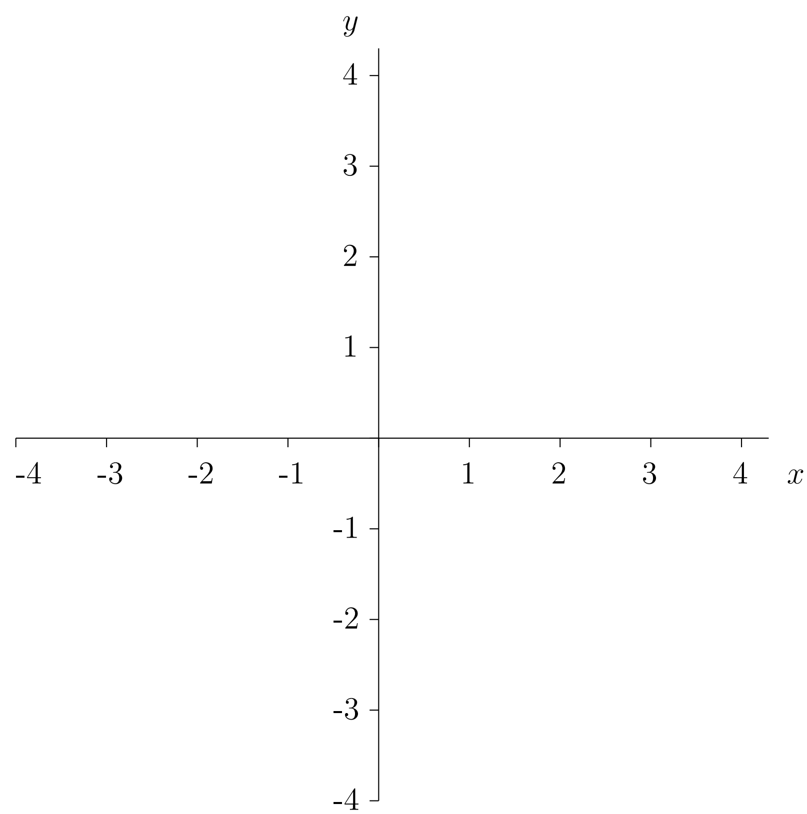
ii) $g(1) = 2, \quad \lim_{x\to 2^+} g(x) = \infty, \qquad \lim_{x\to 2^-} g(x) = -\infty,$

iii) $g'(x) > 0$ when $x < 1$ and $g'(x) < 0$ when $x > 1$ and $x \neq 2$,

iv) $g''(x) > 0$ when $x < -1$ and when $x > 2$,

v) $g''(x) < 0$ when $-1 < x < 2$.

Graph the function $g(x)$ on the axes given below.
Also, provide the information that is asked for below. (You do not need to explain your reasoning here.)



Equation(s) of horizontal asymptote(s) ____________________

Equation(s) of vertical asymptote(s) ____________________

Critical number(s) ____________________

First coordinate(s) of inflection point(s) ____________________

Absolute maximum value of f ____________________

(10) (a) State the Mean Value Theorem.

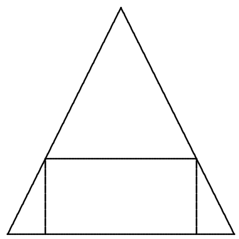
(b) A train is observed traveling at 30 mi/hr at 1:00 p.m. and at 90 mi/hr at 4:00 p.m.. Use the Mean Value Theorem to show that at some time between these two observations the acceleration of the train is 20 mi/hr^2. Your solution should specify the function and the values of the endpoints that you use in the Mean Value Theorem as well as the equation of the Mean Value Theorem.

(11) A rectangle just fits inside an isosceles triangle whose is height is 15 feet and whose base is 6 feet long (as sketched below). Answer the following questions.

(a) Find the height h of the rectangle if b is the length of its base.

(b) Find the height and base of the rectangle of largest area that will fit inside the triangle.

(c) Explain how you know that the area of the rectangle in part (b) is an absolute maximum and not just a local maximum or local minimum.



(a) h as a function of b: ______________________________

(b) $b =$ _______________, $h =$ _______________

(c) Reason: