MA 113 Calculus I Fall 2016
Exam 3 Tuesday, November 15, 2016

Name: ______________________________

Section: ______________________________

Last 4 digits of student ID #: ________

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Free Response Questions: Show your work!

16. (a) State the Mean Value Theorem.

See textbook.

(b) Suppose that f is a differentiable function on the real line and $3 \leq f'(x) \leq 4$ for x in the interval $(2, 7)$. If $f(7) = 9$, use the Mean Value Theorem for f in the interval $[2, 7]$ to determine the largest and smallest possible values for $f(2)$.

By MVT, for some c in $(2,7)$ we have

$$f'(c) = \frac{f(7)-f(2)}{7-2} = \frac{9-f(2)}{5}.$$

So, $3 \leq f'(c) = \frac{9-f(2)}{5} \leq 4$

$\Rightarrow \quad 15 \leq 9 - f(2) \leq 20$

$\Rightarrow \quad -6 \geq f(2) \geq -11.$

Free Response Questions: Show your work!

17. Evaluate the following limits. Be sure to explain your reasoning.

(a) $\lim_{x\to+\infty} x^2 \cdot \sin\frac{\pi}{x^2}$ $= \lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{x^2}\right)}{\frac{1}{x^2}} \underset{L'H}{=} \lim_{x\to\infty} \frac{\cos\left(\frac{\pi}{x^2}\right)\cdot\frac{-2\pi}{x^3}}{\frac{-2}{x^3}}$

$= \lim_{x\to\infty} \cos\left(\frac{\pi}{x^2}\right)\cdot\pi = \pi.$

(b) $\lim_{x\to 0} \frac{e^{3x}-1-3x}{x^2}$ $\underset{L'H,\ \text{form } 0/0}{=} \lim_{x\to 0} \frac{3e^{3x}-3}{2x} \underset{L'H,\ \text{form } 0/0}{=} \lim_{x\to 0} \frac{9e^{3x}}{2} = \frac{9}{2}.$

Free Response Questions: Show your work!

18. Consider the function $f(x) = 3x^4 + 6x^3 - 113$. Use methods of Calculus to solve the following. Be sure to show your work and explain how you obtained your answers.

    (a) Find the interval(s) where the function $f(x)$ is increasing and the interval(s) where the function $f(x)$ is decreasing.

$$f'(x) = 12x^3 + 18x^2 = 6x^2(2x+3)$$

$\Rightarrow$ crit. values are $0, -\frac{3}{2}$.

$-$ $+$ $+$

$-\frac{3}{2}$ 0

f inc on $\left(-\frac{3}{2}, 0\right) \cup (0, \infty)$

f dec on $\left(-\infty, -\frac{3}{2}\right)$.

    (b) Find the interval(s) where the graph of $f(x)$ is concave up and the interval(s) where the graph of $f(x)$ is concave down.

$$f''(x) = 36x^2 + 36x = 36x(x+1).$$

$+$ $-$ $+$

-1 0

f conc. up on $(-\infty, -1) \cup (0, \infty)$

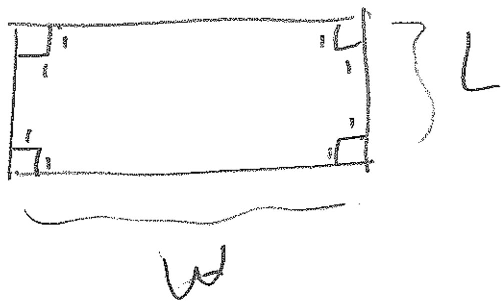
f conc. down on $(-1, 0)$.

Free Response Questions: Show your work!

Name: ______________________ **Student ID Number:** __________

19. A manufacturer wishes to design an open box from a rectangular piece of cardboard having length L and width W. The original piece of cardboard has area 256 cm^2. The manufacturer forms the box by cutting out a square of sidelength 1 cm from each corner and folding up the sides to form the box.

(a) Draw a picture of the box and label all quantities.



$L \cdot W = 256$

(b) Write the equation stating that the area of the cardboard is 256 cm^2.

$L \cdot W = 256.$

(c) Use methods of Calculus to determine what the dimensions of the original piece of cardboard should be in order to produce a box with the maximum volume.

$V = 1 \cdot (L-2)(W-2)$. Since $L = \frac{256}{W}$, we get

$V(w) = \left(\frac{256}{w} - 2\right)(w-2) = 256 + 4 - 2w - \frac{512}{w}$ domain is $(0, \infty)$

$V'(w) = -2 + \frac{512}{w^2} = 0 \Rightarrow w^2 = 256 \Rightarrow w = \sqrt{256}$.

Since $V'(w) > 0$ on $(0, \sqrt{256})$ and $V'(w) < 0$ on $(\sqrt{256}, \infty)$, first der. test for extreme values says max volume is when $W = L = \sqrt{256}$