Exam 3

Form A

Name: ______________________ Section and/or TA: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 12 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **11** (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E) **12** (A) (B) (C) (D) (E)

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Suppose that $\dfrac{dy}{dt} = ky$, where k is a constant and suppose that $y(0) = 2$ and $y(2) = 6$. Find $y(5)$.

   A. $2 \cdot 6^5$

   B. $2 \cdot 6^{5/2}$

   C. $2 \cdot 3^5$

   D. $2 \cdot 3^{5/2}$

   E. None of the above.

2. The half-life of a radioactive substance is 20 years. If a sample has a mass of 100 grams, how much of the sample remains after 50 years?

   A. $100 \cdot e^{\frac{5\ln(2)}{2}}$ grams

   B. $100 \cdot e^{-\ln(5/2)}$ grams

   C. $100 \cdot e^{-5/2}$ grams

   D. $100 \cdot e^{-\frac{5\ln(2)}{2}}$ grams

   E. 12.5 grams

3. Find the rate of change of the volume of a cube with respect to the length of its side s when $s = 9$ meters.

   A. 729 cubic meters of volume per meter of length

   B. 243 cubic meters of volume per meter of length

   C. 81 cubic meters of volume per meter of length

   D. 9 cubic meters of volume per meter of length

   E. None of the above.

4. Two cars leave Lexington at the same time, one traveling due east at 60 miles per hour and one traveling due south at 75 miles per hour. How fast is the distance between them changing after 2 hours?

   A. ≈ 192 mph

   B. ≈ 137 mph

   C. ≈ 68 mph

   D. ≈ 96 mph

   E. ≈ 135 mph

5. Find the average rate of change of the area of a circle with respect to its radius r as r changes from 3 to 8.

   A. 6π

   B. 8π

   C. 11π

   D. 12π

   E. 36π

6. Find the critical points of $f(x) = \dfrac{x^2}{x^2 - 4x + 9}$.

   A. $\{0, -9/2\}$

   B. $\{0, 9/2\}$

   C. $\{0, 3/2\}$

   D. $\{0, 9/4\}$

   E. None of the above.

7. Suppose that $f'(x) = x^2(x+2)(x-2)(x-4)$. Find the interval or intervals where f is decreasing. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

   A. $(-\infty, -2) \cup (2, \infty)$

   B. $(-2, 2) \cup (4, \infty)$

   C. $(-\infty, -2) \cup (2, 4)$

   D. $(2, \infty)$

   E. $(2, 4)$

8. You are given that $f'(x) = x^2(x+2)(x-2)(x-4)$. Find the values of x that give the local maximum and local minimum values of the function $f(x)$. (Read the problem carefully. The given function is $f'(x)$, not $f(x)$.)

   A. Local maximum value of f at $x = 0$ and local minimum values of f at $x = -2, 4$

   B. Local maximum values of f at $x = -2, 4$ and local minimum value of f at $x = 0$

   C. Local maximum value of f at $x = 2$ and local minimum values of f at $x = -2, 4$

   D. Local maximum values of f at $x = -2, 2$ and local minimum values of f at $x = 0, 4$

   E. Local maximum values of f at $x = 0, 4$ and local minimum values of f at $x = -2, 2$

9. Assume that $f''(x) = x(x-2)(x-4)$. Find the points of inflection of the function f. (Read the problem carefully. The given function is $f''(x)$, not $f(x)$.)

   A. $x = 0, 2, 4$

   B. $x = 2$

   C. $x = 4$

   D. $x = 0, 4$

   E. $x = 2, 4$

10. The function $f(x) = e^{|x|}$ has an absolute minimum at $x = 0$ because:

    A. $f'(0) = 0$ and $f''(0) > 0$.

    B. $f'(x) > 0$ for $x > 0$ and $f'(x) < 0$ for $x < 0$, with $f'(0)$ undefined.

    C. $f(x)$ is not differentiable at $x = 0$ and $f''(0) > 0$.

    D. this is the statement of the Mean Value Theorem.

    E. None of the above.

11. Find all values c that satisfy the conditions of the Mean Value Theorem for

$$f(x) = 7x^2 - x + 5$$

on the interval $[a, b] = [-1, 7]$.

A. 3
B. 4
C. $2, 3$
D. $2, 4$
E. None of the above

12. Find two positive numbers whose product is 144 and whose sum is a minimum.

A. 2, 72
B. 3, 48
C. 4, 36
D. 6, 24
E. None of the above

Free Response Questions
Show all of your work

13. Find the following limits

(a) $\lim_{x \to 0} \frac{e^x - 1}{\sin(2x)}$

(b) $\lim_{x \to 0} \frac{1 - \cos(x)}{x^4 + 3x^2}$

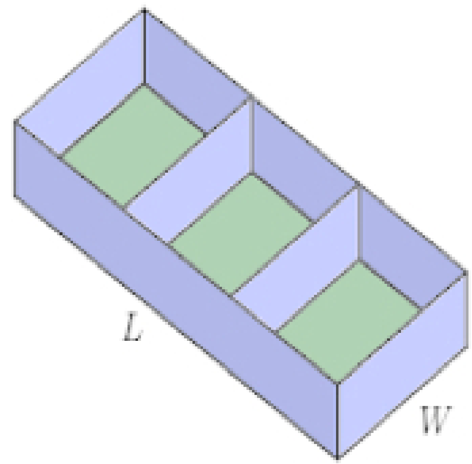
(c) $\lim_{x \to 0} \frac{e^{3x} - e^{-9x}}{\ln(1 + x)}$

14. The volume of a right circular cone of radius r and height h is $V = \frac{\pi}{3}r^2h$. Suppose that the radius and height of the cone are changing with respect to time t.

(a) Find a relationship between $\frac{dV}{dt}$, $\frac{dr}{dt}$, and $\frac{dh}{dt}$.

(b) At a certain instant of time, the radius and height of the cone are 12 in. and 13 in. and are increasing at the rate of 0.2 in/sec and 0.5 in/sec, respectively. How fast is the volume of the cone increasing?

15. Your task is to design a rectangular industrial warehouse consisting of three separate spaces of equal size.



The wall materials cost \$66 per linear foot and your company has allocated \$79200 for those walls.

(a) Find the dimensions which use all of the budget and maximize the total area.

(b) What is the area of the three (equal size) compartments?

16. Let $f(x) = x^4 - 18x^2 - 7$. Be sure to justify each of your answers below.

(a) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

(b) Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

(c) Find the points that give local maximum values of $f(x)$, the points that give local minimum values of $f(x)$, and the points of inflection of $f(x)$.