MA 113 — Calculus I Spring 2008
Alternate Exam 3 April 8, 2008

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. check answers when possible,
2. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
3. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question. Also when appropriately record your answers at the bottom of the page.
You are to answer *two of the last three questions*. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name: ______________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		9
2		8
3		9
4		8
5		9
6		9
7		8
8		9
9		14
10		14
11		14
Free	3	3
		100

(1) Calculate the following limits:

(a)
$$\lim_{x\to\infty} \frac{\sqrt{7+5x+3x^5}}{5+7x+2x^3}.$$

(b)
$$\lim_{x\to\infty} \frac{4+8\sqrt{x}+3x}{4x+8\sqrt{x}+3}.$$

(c)
$$\lim_{x\to\infty} (\sqrt{x^2+x}-x).$$

In each case show the algebraic operations needed to get the correct answer.

(a) ______________

(b) ______________

(c) ______________

(2) Consider the polynomial function

$$f(x) = 2x^3 - 9x^2 + 12x + 5.$$

(a) Find the critical numbers of $f(x)$ and show your work.
(b) Use (a) and either the first or second derivative test to find the local extrema of f.

(a) The critical number(s): ______________

(b) The local maximum: ______________ The local minimum: ______________

(3) Find the absolute maximum value of the function $f(x) = \sin^2 x + \sqrt{3}\cos x$ on the closed interval $[0, \pi]$. Justify your answer.

The absolute maximum value is: ________________________

(4) Consider the polynomial function

$$f(x) = x^4 - 6x^3 + 12x^2 + 10x + 14.$$

(a) Find the interval(s) of concavity of $f(x)$ and show your work.

(b) Find the point(s) of inflection on the graph of f and justify your answer.

(a) $f(x)$ is concave up on: ________________________

$f(x)$ is concave down on: ________________________

(b) The point(s) of inflection $(x, y) =$ ________________________

(5) Find the most general antiderivative for each of the following functions:

(a) $f(x) = x^7 - 15x^4 + 3x - 8$;

(b) $g(x) = 5\sin(x) - 2\cos(x)$;

(c) $h(x) = 4\sqrt[3]{x} + \dfrac{2}{x^2\sqrt{x}}$.

(a) General antiderivative of f: ____________

(b) General antiderivative of g: ____________

(c) General antiderivative of h: ____________

(6) A particle is moving along the x-axis so that its acceleration in $\frac{ft}{s^2}$ is given by $a(t) = 6t + 3$ after t seconds. After 1 second the velocity of the particle is zero, and the particle is 3 feet from the origin in the positive direction after 2 seconds.

(a) Find the velocity $v(t)$ after t seconds.

(b) Find the position $x(t)$ after t seconds.

(a) The velocity in feet per second is $v(t) =$________________________

(b) The position in feet is $x(t) =$________________________________

(7) (a) Sketch the region (in particular, specify the function and the interval you are using), whose area is given by $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{(1+\frac{i}{n})^2}$.

(b) Calculate the following definite integral by first interpreting it using areas and then applying area formulas from geometry: $\int_0^1 (3x-2)dx$.

(a) Function used: ____________ Interval: ____________

(b) ____________

(8) Find the two nonnegative numbers x and y whose sum is 9 such that x^2y+24x is maximal. Carefully justify your answer.

$x =$ ____________ $y =$ ____________

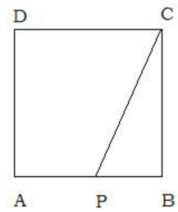
Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(9) (a) State the Mean Value Theorem. Use complete sentences.

(b) Is the Mean Value Theorem applicable to the function $f(x) = |x^2 - 9|$ on the interval $[0, 4]$? Justify your answer.

(c) Use the Mean Value Theorem to explain why one should not drive 9 miles in 15 minutes on a street where the speed limit is 35 miles per hour.

(10) Let $ABCD$ be a square of sidelength 1 mile. Find a point P on the side AB such that the time traveling from A to C via the line segments AP and PC is minimized, assuming the speed limit on AP is 1/2 mile per minute and on PC is 1/3 mile per minute (see the sketch). Specify the point P by giving its distance to B. Carefully justify why your answer is correct by using calculus.



The distance $|PB| =$____________ miles.

(11) Sketch the graph of a function $f(x)$ defined for $x > 0$ such that

(a) $\lim_{x \to 0^+} f(x) = 3$,

(b) $f(2) = f(4) = 2$, $f(3) = 4$,

(c) $\lim_{x \to \infty} f(x) = f(1) = 1$,

(d) $f''(x)$ exists and is continuous for all $x > 0$,

(e) $f'(1) = f'(3) = f''(2) = f''(4) = 0$, and $f'(x)$ and $f''(x)$ are not zero for all other values of x.

Label all the important points on the graph by their x- and y-values and sketch the graph such that the intervals of increase/decrease and of the same concavity can be read off.