MA 113 — Calculus I Spring 2009
Exam 3 April 14, 2009

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number on the table below. You may also obtain up to 10 extra credit points by correctly answering the five true-false questions on the last page (note that 1 extra credit point will be deducted for each incorrect answer on these questions).

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ______________________

Section: ________

Last four digits of student identification number: ________

Question	Score	Total
1		10
2		7
3		10
4		12
5		10
6		7
7		9
8		16
9		16
10		16
EC		10
Free	3	3
		100

(1) Consider the function $f(x) = 2x^3 - 3x^2 - 36x + 4$ on the interval $(-\infty, \infty)$.

(a) Find the critical number(s) of f.

(b) Find the interval(s) of increase and decrease for f.

(c) Find the local extrema of f.

(a) The critical number(s): ________________

(b) Interval(s) of increase: ________________ and decrease: ________________

(c) The local maximum is at $x =$ ________________ and $f(x) =$ ________________
The local minimum is at $x =$ ________________ and $f(x) =$ ________________

(2) Find the absolute minimum value of the function

$$f(t) = t + \sqrt{1 - t^2}$$

on the interval $[-1, 1]$. Be sure to also specify the value of t where the absolute minimum is achieved.

Absolute minimum _______________ at $t =$ _______________

(3) Consider the function

$$f(x) = 2x + \sin x$$

on the interval $(-\pi, 2\pi)$.

(a) Find the interval(s) of concavity of the graph of $f(x)$; show your work.

(b) Find the point(s) of inflection of the graph of $f(x)$; justify your work.

(a) Interval(s) where the graph is concave up: ____________________

Interval(s) where the graph is concave down: ____________________

(b) Point(s) of inflection (x and y coordinates): ____________________

(4) (a) Let $f(x) = \sqrt{16 + x}$. First, find the linear approximation to $f(x)$ at $x = 0$. Then use the linear approximation to estimate $\sqrt{15.75}$. Present your solution as a rational number.

(b) Suppose that g is a differentiable function whose tangent line at $x = 2$ is given by $y = 2x - 1$. Suppose that we are given an initial approximation $x_1 = 2$ to a root c of $g(x)$. Starting with the initial approximation $x_1 = 2$, use Newton's method to find a new approximation x_2 to c.

(a) Linear approximation $L(x) =$ ______________________________
$\sqrt{15.75} \approx$ ______________________________

(b) $x_2 =$ ______________________________

(5) Find the general antiderivative of each function:

(a) $f(x) = x^2 + 1.$

(b) $g(\theta) = \dfrac{1}{\theta^2} + 2\cos\theta.$

(c) $h(t) = 2t^{1/2} + e^t.$

(a) ____________________

(b) ____________________

(c) ____________________

(6) Use l'Hospital's rule to evaluate the following limits:

(a) $\lim_{x \to 0} \frac{\tan(\pi x)}{x}$.

(b) $\lim_{x \to \infty} e^{-x} \ln(x + 3)$.

(a) $\lim_{x \to 0} \frac{\tan(\pi x)}{x} =$ ________________________

(b) $\lim_{x \to \infty} e^{-x} \ln(x + 3) =$ ________________________

(7) Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{2x^3 + \sqrt{x^4 + 2}}{x^3 + 1}.$$

Be sure to compute all limits that are needed to support your answer.

Equation(s) of horizontal asymptote(s): ______________________

Equation(s) of vertical asymptote(s): ______________________

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) State the Mean Value Theorem. Use complete sentences.

(b) Assume that f and g are differentiable functions on $(-\infty, \infty)$, that $f'(x) = g'(x)$ for all x, that $f(x) = \sin x$, and that $g(0) = 1$. What is $g(x)$?

(c) Suppose that g is differentiable for all x and that $-5 \leq g'(x) \leq 2$ for all x. Assume also that $g(0) = 2$. Based on this information, is it possible that $g(2) = 8$? Use the Mean Value Theorem to justify your answer.

(b) $g(x) =$ ______________________ (c) yes / no (*circle the correct answer*)

(9) In parts (b) and (c), be sure to fully justify all steps you take to compute the given limits.

(a) State l'Hospital's Rule. Use complete sentences.

(b) Find $\lim_{x \to 1} \frac{x-1}{\ln(x+1)}$.

(c) Find $\lim_{x \to 0^+} x^x$.

(b) $\lim_{x \to 1} \frac{x-1}{\ln(x+1)} =$ ______________ (c) $\lim_{x \to 0^+} x^x =$ ______________

(10) Find the point(s) on the hyperbola $y = \frac{16}{x}$ that is (are) closest to the point $(0, 0)$. Be sure to state clearly what function you choose to minimize or maximize and why.

Point(s) $(x, y) =$ ____________________

Extra Credit Problem.

Mark the correct answers below. For each correct answer you earn 2 points, and for each incorrect answer 1 point will be subtracted. Therefore, **it might be wise to skip a question rather than risking losing a point.** However, your final score on this problem will not be negative! You need not justify your answer.

True	**False**	**Statement**
☐	☐	If $f'(c) = 0$ and f' changes from negative to positive at c, then f must have an absolute minimum at c.
☐	☐	There exists a function $f(x)$ such that $f(x) < 0$, $f'(x) > 0$, and $f''(x) < 0$ for all x.
☐	☐	Suppose that a differentiable function f is increasing, and $f(x) > 0$ for all x. Then $g(x) = \dfrac{1}{f(x)}$ is also an increasing function.
☐	☐	If f is odd, then f' is also odd.
☐	☐	If the function f is differentiable, $f(1) = 1$, and $f(3) = -3$, then there exists a number c between 1 and 3 such that $f'(c) = -2$.