Last 4 digits of student ID #:_____________________

This is a two-hour exam. This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.

- Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),

- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.
Free Response Questions: Show your work!

15. (a) Find the value of \( \lim_{x \to 0} \frac{\sin 6x}{\tan 7x} \)

\[ \lim_{x \to 0} \frac{\sin (6x)}{\tan (7x)} = \lim_{x \to 0} \frac{6 \cos (6x)}{7 \sec^2 (7x)} = \frac{6 \cos (0)}{7 \sec^2 (0)} = \frac{6}{7} \]

\[ \begin{align*}
\text{+1 derivative of top} \\
\text{+1 plug in 0} \\
\text{+1 final answer}
\end{align*} \]

(b) If \( f''(x) = 3x + 1 \) where \( f(0) = 2 \) and \( f(1) = 2 \), find \( f(x) \).

\[ f''(x) = \frac{d^2}{dx^2} \left( \frac{3}{2} x^2 + \frac{1}{2} x + C \right) = \frac{3}{2} x + C x + d \]

\[ f(x) = \frac{1}{3} \cdot \frac{3}{2} x^3 + \frac{1}{2} x^2 + C x + d = \frac{1}{2} x^3 + \frac{1}{2} x^2 + C x + d \]

\[ f(0) = \frac{1}{2} (0)^3 + \frac{1}{2} (0)^2 + C (0) + d = d = 2 \] \( \text{+1 solve for } d \)

\[ f(1) = \frac{1}{2} (1)^3 + \frac{1}{2} (1)^2 + C (1) + d = 1 + C + d = C + 3 = 2 \]

\[ C = -1 \] \( \text{+1 solve for } C \)

\[ f(x) = \frac{1}{2} x^3 + \frac{1}{2} x^2 - 1x + 2 \] \( \text{+1 final answer} \)
Free Response Questions: Show your work!

14. Find the point(s) on the hyperbola \( y = \frac{9}{x} \) that is (are) closest to \((0,0)\). (Hint: minimize the square of an appropriate distance function.)

Minimize

\[ D = (x-0)^2 + (y-0)^2 = x^2 + y^2. \]

subject to \((x,y)\) is on the hyperbola, that is,

\[ y = \frac{9}{x}. \]

Minimize (change to a function of one variable).

\[ D = x^2 + y^2 = x^2 + \left(\frac{9}{x}\right)^2 = x^2 + \frac{81}{x^2}. \]

Differentiate

\[ \frac{dD}{dx} = 2x - \frac{81}{x^3}. \]

Critical points:

Set derivative = zero.

\[ 2x - \frac{81}{x^3} = 0 \]

\[ 2x = \frac{81}{x^3} \]

\[ x^4 = 81 = 3^4 \]

\[ \Rightarrow x = 3, -3. \]

Thus, min. distance is at \((3, \frac{9}{3})\) and \((-3, -\frac{9}{3})\).
Free Response Questions: Show your work!

13. Recall that \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).

(a) Find the value of \( \sum_{i=1}^{12} (5i + 2) \).

\[
= 5 \cdot \sum_{i=1}^{12} i + 2 \sum_{i=1}^{12} 1
\]

\[
= 5 \cdot \frac{12 \cdot 13}{2} + 2 \cdot 12
\]

\[
= 5 \cdot 6 \cdot 13 + 24
\]

\[
= 414
\]

+1 for attempting problem.

-1 Arithmetic error

-1 linearity of sum

-1 plug into formula

-1 adding 2 incorrectly

(b) Calculate \( L_3 \) for \( f(x) = 2 + x^2 \) over the interval \([0, 3]\).

\[
\Delta x = \frac{3-0}{3} = 1, \quad x_0 = 0, \quad x_1 = 1, \quad x_2 = 2 \quad \text{so}
\]

\[
L_3 = \sum_{i=0}^{2} f(x_i) \cdot \Delta x = (2+0^2) \cdot 1 + (2+1^2) \cdot 1 + (2+2^2) \cdot 1
\]

\[
= 2 + 3 + 6
\]

\[
= 11
\]

+1 for attempting problem

-1 for using \( R_3 \)

-1 for using wrong \( f(x) \)

-1 for adding all 4 endpoints
Free Response Questions: Show your work!

16. This problem concerns the definition of the derivative using limits.

(a) State the Mean Value Theorem.  

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

If \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\), then there exists \( c \in (a, b) \) such that.

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

Equivalent expression is okay.

(b) Suppose that \( g(x) \) is differentiable for all \( x \) and that \(-2 \leq g'(x) \leq 4\) for all \( x \). Also assume that \( g(1) = 3 \). Find the largest possible value for \( g(3) \).

\[
\text{Set up: } \quad -2 \leq \frac{g(3) - g(1)}{3 - 1} \leq 4
\]

\[
\Rightarrow \quad -2 \leq \frac{g(3) - 3}{2} \leq 4
\]

\[
\Rightarrow \quad -4 \leq g(3) - 3 \leq 8
\]

\[
\Rightarrow \quad -1 \leq g(3) \leq 11
\]

The largest possible value for \( g(3) \) is 11.