Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. Please:

1. check answers when possible,

2. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,

3. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

You are to answer two of the last three questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Notice that there is an additional bonus problem. For its solution you can earn up to 10 points for extra credit.

Name: ———————————————————

Section: —————————

Last four digits of student identification number: —————————

Question	Score	Total
1		6
2		6
3		7
4		9
5		9
6		8
7		7
8		9
9		8
Extra Credit		10
10		14
11		14
12		14
Free	3	3
Total		

(1) Determine the following limits.

(a)

$$\lim_{x \to \infty} \frac{4x^3 + x^2 + \sqrt{x}}{1 - x^3}.$$

(b)

$$\lim_{x \to 3} \frac{x^2 - 8x + 15}{x^2 - 9}.$$

(a) **Answer to Q.1(a)** _____

(b) **Answer to Q.1(b)** _____

(2) Find the equation of the tangent line to the curve:

$$x^2 + xy + y^2 = 7 \text{ at the point } P(1, 2).$$

Be sure to simplify your final answer in the form $y = mx + b$.

The equation of the tangent line is

$$y = \underline{\hspace{6cm}}.$$

(3) Let f be a function defined on the interval $(0, \infty)$ by

$$f(x) = \begin{cases} \pi \sin(x) & \text{if } 0 < x < \frac{\pi}{6} \\ 2x + c & \text{if } \frac{\pi}{6} \leq x \end{cases}$$

where c is a real number.

(a) Determine a value of c such that f is continuous on $(0, \infty)$, if such c exists.

(b) Determine where the resulting function is differentiable.

(a) **Answer Q.3(a):** $c =$ _____

(b) **Answer Q.3(b):** Intervals where f is differentiable are: _____

(4) Find the derivatives of the following functions:

(a)
$$f(x) = \sqrt{x^3 + 4x^2 + 4}.$$

(b)
$$g(x) = \frac{\sin(x^2)}{x^2}.$$

(c)
$$h(x) = \int_0^x \left(\frac{(1 - t^2)^3}{\sqrt{t^4 + t}} \right) dt.$$

(a) **Answer to Q.4(a)** $f'(x) =$ _____

(b) **Answer to Q.4(b)** $g'(x) =$ _____

(c) **Answer to Q.4(c)** $h'(x) =$ _____

(5) Consider the function f defined on the interval $[-3, 3]$ by the formula

$$f(x) = x^3 - 3x^2 + 14.$$

(a) Determine all the critical numbers for f in the open interval $(-3, 3)$.

(b) Determine the locations of all the local extrema of f on the closed interval $[-3, 3]$.

(c) What are the absolute extremum values of f on $[-3, 3]$. Determine all the corresponding values of x where these are attained.

(a) **Critical numbers for f in $(-3, 3)$:** _____

(b) **Local maxima of f on $[-3, 3]$ are at:** _____ **Local minima are at:** _____

(c) **Absolute maximum value of f is** _____ **It is attained at** _____

Absolute minimum value of f is _____ **It is attained at** _____

(6) Consider the function f defined by

$$f(x) = \int_0^x \frac{dt}{t^2 + t + 1}.$$

(a) Determine the first two derivatives $f'(x)$ and $f''(x)$.

(b) Using your results in (a) or other arguments, determine the interval(s) on which f is an increasing function.

(c) Using your results in (a) or other arguments, determine the interval(s) on which the graph of $f(x)$ is concave upward.
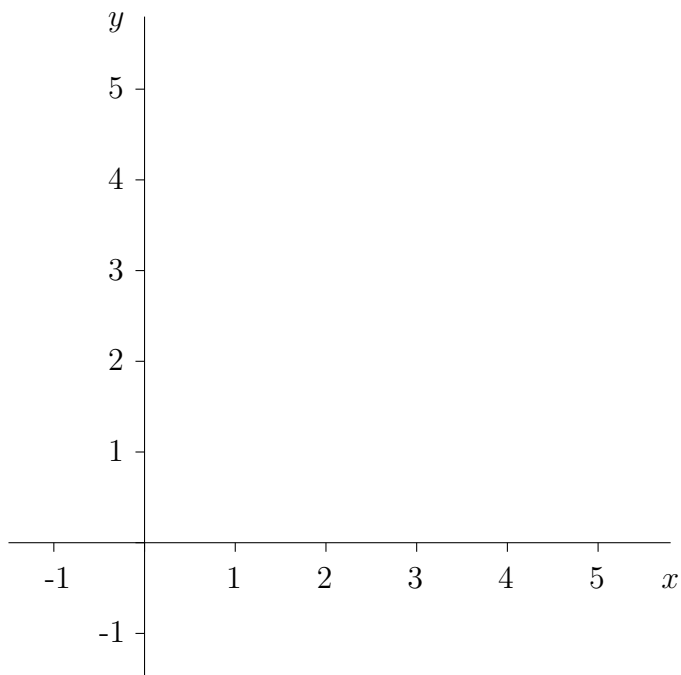
(a) $\mathbf{f'(x)} = $ _____ , $\mathbf{f''(x)} = $ _____

(b) **Interval(s) where f is increasing:** _____

(c) **Interval(s) where the graph of f is concave upward:** _____

(7) Sketch the region enclosed between the graphs of the two curves:

$$y = (x - 2)^2 \text{ and } y = x$$

and compute its area.

The area of the region is: _____

(8) Calculate the following integrals. For indefinite integrals, be sure to add the arbitrary constant c.

(a)
$$\int 6x^2(1 + 2x^3)^4 \, dx.$$

(b)
$$\int \frac{t}{(4t^2 + 5)^2} \, dt.$$

(c)
$$\int_0^{\pi/6} \sin(3x) \cos^2(3x) \, dx.$$

(a) **Answer to Q.8(a)** _____

(b) **Answer to Q.8(b)** _____

(c) **Answer to Q.8(c)** _____

(9) Consider the integral

$$\int_1^2 \frac{1}{x}\, dx.$$

Answer the following questions.

(a) Estimate the integral by the Riemann sum obtained by using a subdivision into three equal parts and using the right endpoints as sample points.
Be sure to write all the details and to write the final answer as a single fraction.

(b) Determine if your estimate is less than or bigger than the actual value of the integral. Explain.

(a) **Answer to Q.9(a)** _____

(b) **Answer to Q.9(b)** _____

Extra Credit Problem.

Determine if the following statements are true or false. No justification is required.

(a) Suppose the function f is continuous on $[1, 5]$ and differentiable on $(1, 5)$.
If $f(1) = -5$ and $f(5) = 20$, then $f'(t) \geq 5$ for at least one t in $(1, 5)$.

Answer: True ☐ **False** ☐

(b) Suppose the function f is continuous on the set of real numbers.
Then f is differentiable.

Answer: True ☐ **False** ☐

(c) Suppose that f is a continuous function on $[1, 5]$ and $f'(2) = 0$. Then f has a local extremum at 2.

Answer: True ☐ **False** ☐

(d) Let f be a function that is twice differentiable on the set of real numbers.
If $f''(2) = 0$ then f has an inflection point at 2.

Answer: True ☐ **False** ☐

(e) If f is a continuous function on the closed interval $[1, 10]$, then f has an absolute maximum at some point c in $[1, 10]$.

Answer: True ☐ **False** ☐

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(10) (a) Carefully state the Fundamental Theorem of Calculus (both parts).

(b) Consider the function f defined by $f(x) = \int_1^x \frac{dt}{1+t^2}$. Determine the derivatives of $f(x)$ and $h(x) = f(\frac{1}{x})$.

(c) Consider the function g on $(0, \infty)$ defined by $g(x) = f(x) + f(\frac{1}{x})$. Using your results in (b) or other arguments, argue that g is a constant function.

(d) Determine $g(1)$.

Derivative of $f(x)$: _____

Derivative of $f(\frac{1}{x})$: _____

$g(1) =$ _____

(11) Consider the quadrangle whose vertices are $A(0, 1)$, $B(1, 1)$, $C(1, 3)$, $D(0, 2)$.

    (a) Make a rough sketch of the quadrangle.

    (b) Set up the integral to determine the volume obtained by rotating the quadrangle about the x-axis.

    (c) Evaluate the integral and determine the volume.

    (d) If the same quadrangle is rotated about the line $x = -3$, write down the integral that computes the volume of the resulting solid of revolution. You are not asked to evaluate this new integral.

        (c) **Volume:** _____

        (d) **Integral:**_____

(12) A company wishes to manufacture a strong box with a square base and a volume of 625 cubic inches.

It is given that the sides of the box cost \$1 per square inch to manufacture and the top and the bottom cost \$5 per square inch to manufacture.

Find the dimensions and the cost of the cheapest such box.

Height of the cheapest box: ———————————

Base side length of the cheapest box: ———————————

Cost of the cheapest box: ———————————