

Answer all of questions 1–7 and choose two of questions 8–10 to answer. Please indicate which of problems 8–10 is not to be graded by crossing through its number on the table below. At the end of the exam you find an Extra Credit Problem. This problem is optional and might earn you up to 10 bonus points.

Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. All other electronic devices including pagers and cell phones should be in the off position for the duration of the exam. Please:

1. **Clearly indicate your answer and the reasoning used to arrive at that answer. Unsupported answers may not receive credit. You need not justify your answers to the Extra Credit Problem.**
2. **Give exact answers, rather than decimal approximations to the answer.**

Each question is followed by space to write your answer. Please write your solutions neatly in the space provided.

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Last four digits of student identification number: \_\_\_\_\_

Question	Score	Total
1		7
2		9
3		10
4		12
5		10
6		12
7		10
8		15
9		15
10		15
<b>Total</b>		<b>100</b>
Extra Credit		10

- (1) Use calculus to find the absolute maximum and minimum values of  $f(x) = xe^{-3x}$  on  $[0,1]$ . Also list the points where these extreme values occur.

Absolute maximum value = \_\_\_\_\_ at  $x =$  \_\_\_\_\_

Absolute minimum value = \_\_\_\_\_ at  $x =$  \_\_\_\_\_.

(2) Given the function  $f$  defined for all  $x$  by

$$f(x) = \begin{cases} 2x + 1 & \text{for } x < 3 \\ x^2 + Bx + 10 & \text{for } x \geq 3 \end{cases}$$

- (a) Find  $B$  so that  $f$  is continuous for all  $x$ . **Show all limits that are needed to support your answer.**
- (b) Use the value for  $B$  you found in (a) and determine all points  $x$  at which  $f$  is differentiable. **Indicate your reasoning.**

(a)  $B =$  \_\_\_\_\_

(b)  $f$  is differentiable on the interval(s) \_\_\_\_\_

(3) Find the following limits. **Justify your steps in finding each limit.**

(a)  $\lim_{t \rightarrow 0} \frac{2 + 6 \sin(7t)}{2t + 3} = \underline{\hspace{2cm}}$ .

(b)  $\lim_{x \rightarrow -\infty} x e^x = \underline{\hspace{2cm}}$ .

(c)  $\lim_{t \rightarrow 0} \frac{1 - \cos(t)}{t^2} = \underline{\hspace{2cm}}$ .

(4) Find the following derivatives. **Show your work!**

(a)  $g'(x)$  when  $g(x) = x^2 \tan(2x)$ .

(b)  $\frac{dy}{dx}$ , where  $1 + xy^2 + x^3 = \sin(y)$ . Your answer should be an expression involving  $x$  and  $y$ .

(c)  $f''(x)$  when  $f(x) = \frac{4}{\sqrt{3 + 5x}}$ .

(a)  $g'(x) =$  \_\_\_\_\_

(b)  $\frac{dy}{dx} =$  \_\_\_\_\_

(c)  $f''(x) =$  \_\_\_\_\_

(5) Consider the function  $f(x) = \frac{x+2}{x-1}$ .

(a) Compute the Riemann sum for  $f$  on the interval  $[2, 6]$  with  $n = 4$  subintervals and the right endpoints as sample points.

(b) Show that  $f$  is decreasing on the interval  $[2, 6]$ .

(c) Use (b) to determine whether the Riemann sum in (a) is greater or less than the actual value of the integral  $\int_2^6 \frac{x+2}{x-1} dx$ . **Do not compute this integral!**

(a) Riemann sum = \_\_\_\_\_

(c) Riemann sum is \_\_\_\_\_ than the integral.

(6) Find the following integrals. **Show your work.**

(a)  $\int (10x^4 - 3x^2 + 4e^x - 7\sin(x)) dx =$  \_\_\_\_\_

(b)  $\int_1^3 x^2 e^{x^3} dx =$  \_\_\_\_\_

(c)  $\int_1^x \frac{t^2 + 3}{t} dt =$  \_\_\_\_\_

- (7) A particle is traveling along a straight line so that its velocity at time  $t$  is given by  $v(t) = 4t - t^2$  (measured in meters per second).
- (a) Graph the function  $v(t)$ .
  - (b) Find the total distance traveled by the particle during the time period  $0 \leq t \leq 5$ .

Total distance traveled = \_\_\_\_\_

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) Let  $f$  be a function on the interval  $[a, b]$ . State the first part of the fundamental theorem of calculus. Do not forget to include the assumptions needed for  $f$ .

(b) Illustrate your answer in (a) by finding the derivative of  $F(x) = \int_1^x (\ln t)^2 dt$  at  $x = e$ .

(c) Find the derivative of the function  $g(x) = \int_{x^2}^4 \sqrt{3 + 5t^4} dt$ .

(b)  $F'(e) =$  \_\_\_\_\_

(c)  $g'(x) =$  \_\_\_\_\_

- (9) A poster is to be made out of  $150 \text{ in}^2$  cardboard and has to have 1-inch margins at the bottom and the two sides and a 2-inch margin at the top.
- (a) Among all such posters what dimensions (width and height of the cardboard) will give the largest printed area?
- (b) What is the largest printed area?

(a) Dimensions of the poster = \_\_\_\_\_ in.

(b) Largest printed area = \_\_\_\_\_  $\text{in}^2$ .

(10) For this problem use the information that for a sphere with radius  $r$

the surface area is  $4\pi r^2$  and the volume is  $\frac{4}{3}\pi r^3$ .

A spherical snowball melts so that its surface area decreases at a rate of  $1 \text{ cm}^2/\text{min}$ .

- (a) Find the rate at which the radius decreases at the moment when the radius is 3 cm.
- (b) Find the rate at which the volume decreases at the moment when the radius is 3 cm.

(a) Radius decreases at a rate of \_\_\_\_\_  $\text{cm}/\text{min}$ .

(b) Volume decreases at a rate of \_\_\_\_\_  $\text{cm}^3/\text{min}$ .

**Extra Credit Problem.**

Circle the correct answer. For each correct answer you earn 2 points, and for each incorrect answer 1 point will be subtracted. Therefore, **it might be wise to skip a question rather than risking to lose a point.** However, your final score on this problem will not be negative! You need not justify your answer.

True	False	Statement
<input type="checkbox"/>	<input type="checkbox"/>	$\frac{d}{dx}(5^x) = x5^{x-1}.$
<input type="checkbox"/>	<input type="checkbox"/>	The function $f(x) = \int_1^x  2t - 1  dt$ is differentiable on $(-\infty, \infty).$
<input type="checkbox"/>	<input type="checkbox"/>	If the function $f$ is continuous on the interval $[a, b]$ , then $f$ is differentiable on the interval $(a, b).$
<input type="checkbox"/>	<input type="checkbox"/>	If the function $f$ is differentiable and $f(-1) = f(1)$ , then there exists a number $c$ such that $ c  < 1$ and $f'(c) = 0.$
<input type="checkbox"/>	<input type="checkbox"/>	$\int_{-\pi}^{\pi} \sin(x^3) dx = 0.$