

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number in the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: _____

Section: _____

Last four digits of student identification number: _____

| Question | Score | Total |
|----------|-------|-------|
| 1 | | 9 |
| 2 | | 13 |
| 3 | | 8 |
| 4 | | 6 |
| 5 | | 12 |
| 6 | | 9 |
| 7 | | 10 |
| 8 | | 15 |
| 9 | | 15 |
| 10 | | 15 |
| Free | 3 | 3 |
| | | 100 |

(1) Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin(2x)}{5x}$

(b) $\lim_{x \rightarrow \infty} \frac{\ln(2 + e^{2x})}{5x}$

(c) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\tan x}$

(a) $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin(2x)}{5x} = \underline{\hspace{2cm}}$

(b) $\lim_{x \rightarrow \infty} \frac{\ln(2 + e^{2x})}{5x} = \underline{\hspace{2cm}}$

(c) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\tan x} = \underline{\hspace{2cm}}$

(2) Consider the function $g(x) = x^3 - 6x^2 + 9x + 2$ on the closed interval $[-1, 4]$.

(a) List all of the critical points of $g(x)$.

(b) On what intervals is $g(x)$ increasing?

(c) On what intervals is $g(x)$ concave down?

(d) List all x - and y -coordinates for the absolute maximum.

(e) List all x - and y -coordinates for the absolute minimum.

(a) Critical points: $x =$ _____

(b) $g(x)$ is increasing on _____

(c) $g(x)$ is concave down on _____

(d) The absolute maximum occurs at the point(s): _____

(e) The absolute minimum occurs at the point(s): _____

(3) Consider the curve $y^2 + xe^y = 1$.

(a) Find the derivative, $\frac{dy}{dx}$, of y .

(b) Find the slope of the tangent line to this curve at the point $(1, 0)$.

(c) Find the equation of the tangent line at the point $(1, 0)$. Express it in the form $y = mx + b$.

(a) $\frac{dy}{dx} =$ _____

(b) The slope of the tangent line: $m =$ _____

(c) The equation of the tangent line: _____

(4) (a) If $f(x) = x^{10}g(x)$, $g(1) = 2$, and $g'(1) = 3$, find $f'(1)$.

(b) If $h(x) = \cos(x^2 + e^{x^2})$, find $h'(x)$.

(a) $f'(1) =$ _____

(b) $h'(x) =$ _____

(5) Find the following integrals and/or antiderivatives. You must show all of your work to receive full credit. An answer without supporting work will receive no credit.

(a) $\int_1^2 \left(10x^4 - 4x^3 - 2x - \frac{2}{x^2} + 2 \right) dx$

(b) $\int_1^x \frac{t^2 \cos t + 4}{t^2} dt$

(c) $\int x \sin(x^2) dx$

(d) $\int_0^4 \frac{x}{x+9} dx.$

(a) $\int_1^2 \left(10x^4 - 4x^3 - 2x - \frac{2}{x^2} + 2 \right) dx = \underline{\hspace{2cm}}$

(b) $\int_1^x \frac{t^2 \cos t + 4}{t^2} dt = \underline{\hspace{2cm}}$

(c) $\int x \sin(x^2) dx = \underline{\hspace{2cm}}$

(d) $\int_0^4 \frac{x}{x+9} dx = \underline{\hspace{2cm}}$

(6) Consider the function $F(x) = \int_0^x \frac{t}{1+t^4} dt$.

(a) Find all intervals on which $F(x)$ is increasing.

(b) Find all intervals on which $F(x)$ is concave up.

(a) $F(x)$ is increasing on _____

(b) $F(x)$ is concave up on _____

(7) For this problem use the following information about a basketball.

$$\text{Surface Area: } A = 4\pi r^2 \quad \text{Volume: } V = \frac{4}{3}\pi r^3.$$

A basketball is being inflated and its volume is increasing at the rate of $5 \text{ cm}^3/\text{sec}$. Be sure to remember to include units in your response.

(a) Find the rate at which the radius, r , of the ball is changing when the radius is 5 cm.

(b) Is the radius increasing or decreasing when the radius is 5 cm? Justify your answer.

(c) Find the rate at which the surface area is changing when the radius is 5 cm.

(d) Is the surface area increasing or decreasing when the radius is 5 cm? Justify your answer.

Work two of the following three problems. Indicate the problems that is not to be graded by crossing through its number on the front of the exam.

(8) (a) State the Mean Value Theorem. Use complete sentences.

(b) Determine the values for the constants a and b such that the function f defined by

$$f(x) = \begin{cases} 1 & x = 0 \\ ax + b & 0 < x \leq 1 \\ x^2 + 4x + 2 & 1 < x \leq 3 \end{cases}$$

satisfies all of the hypotheses of the Mean Value Theorem on the interval $[0, 3]$. **As usual, show your work to support your answer.**

(c) Find at least one point in $[0, 3]$ where the conclusion of the theorem is satisfied.

(9) (a) State both parts of the Fundamental Theorem of Calculus. Use complete sentences.

(b) Consider the function f on $[1, \infty)$ defined by $f(x) = \int_1^x \sin^2(u^2) du$. Explain why the function $f(x)$ is increasing.

(c) Find the derivative of the function $g(x) = \int_{x^3}^1 \sin^2(u^2) du$.

(10) A particle moves along the x -axis so that its velocity at any time $t \geq 0$ is given by

$$v(t) = 3t^2 - 2t - 1 \text{ miles/minute.}$$

The position $x(t)$ is 5 for $t = 2$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.