MA 113 — Calculus I Fall 2012
Exam 4 12-12-12

Name: ____________________

Section: ____________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	~~D~~	E
2	A	B	~~C~~	D	E
3	A	B	~~C~~	D	E
4	A	B	C	D	~~E~~
5	A	B	C	~~D~~	E
6	~~A~~	B	C	D	E
7	A	B	~~C~~	D	E
8	A	~~B~~	C	D	E
9	A	B	C	~~D~~	E
10	A	B	C	~~D~~	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Which of the following is an equation of the tangent line to the curve given by

$$2x^2y - e^xy^3 - 8x + 1 = 0$$

at the point $(0, 1)$.

A. $y = x + 3.$

B. $y = x + 1.$

C. $y = -3x.$

D. $y = -3x + 1.$

E. $y = -3x + 3.$

$4xy + 2x^2y' - e^xy^3 - 3e^xy^2y' - 8 = 0$

@ $(0,1)$

$-1 - 3y' - 8 = 0$

$y' = -3$

$y - 1 = -3x$

$\boxed{y = -3x + 1}$

2. Let $c > 0$. Consider the function

$$f(x) = c\ln(x) - 8x^2$$

on the interval $(0, \infty)$. Find the maximal open interval on which f is decreasing.

A. $(0, \infty)$.

B. $(0, \frac{\sqrt{c}}{4})$.

C. $(\frac{\sqrt{c}}{4}, \infty)$.

D. $(\frac{c}{16}, \infty)$.

E. The function is nowhere decreasing.

$f'(x) = \frac{c}{x} - 16x = 0$

$\Rightarrow x^2 = \frac{c}{16}$, thus $x = \frac{\sqrt{c}}{4}$

	0	$\frac{\sqrt{c}}{4}$
$f'(x)$	$+$	$-$
$f(x)$	$\nearrow$	$\searrow$

Record the correct answer to the following problems on the front page of this exam.

3. Consider the function

$$f(x) = x^2 + 2x.$$

Compute the Riemann sum for f on the interval $[0, 2]$ with $n = 4$ subintervals of equal length and the left endpoints as sample points.

A. $\frac{3}{4}$

B. $\frac{11}{4}$

C. $\frac{19}{4}$

D. $\frac{27}{4}$

E. $\frac{35}{4}$

$0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2$

$\Delta = \frac{1}{2}$

$\frac{1}{2}\left(f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2})\right)$

$= \frac{1}{2}\left(0 + \frac{5}{4} + 3 + \frac{21}{4}\right) = \frac{19}{4}$

4. Determine the constant k such that

$$\underbrace{\int_0^{\frac{1}{k}} e^{kx}\,dx}_{\parallel} = 2.$$

A. $\frac{e+1}{2}$

B. $\frac{e}{2}$

C. $\frac{1}{2}$

D. $\frac{2}{e-1}$

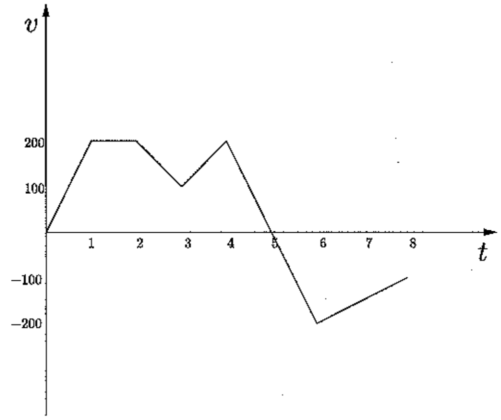
E. $\frac{e-1}{2}$

$\frac{1}{k} e^{kx}\Big|_0^{\frac{1}{k}} = \frac{1}{k}(e-1) \stackrel{!}{=} 2$

Thus $k = \frac{e-1}{2}$

Record the correct answer to the following problems on the front page of this exam.

5. A squirrel runs along a phone line. Its velocity (in meters/minute) at time t (in minutes) during the time interval $[0, 8]$ is shown in the following picture.



At what time(s) does the squirrel change its direction?

A. At times 1 and 2.

B. At times 3, 4 and 6.

C. Only at time 4.

D. Only at time 5.

E. Only at time 6.

6. Let c be a positive constant. Determine $\int_0^c \frac{dx}{x^2+c}$.

(Note that $\arctan(x)$ and $\arcsin(x)$ are also denoted by $\tan^{-1}(x)$ and $\sin^{-1}(x)$, respectively.)

A. $\frac{\arctan(\sqrt{c})}{\sqrt{c}}$

B. $\frac{\arcsin(\sqrt{c})}{\sqrt{c}}$

C. $\sqrt{c}\arctan(\sqrt{c})$

D. $c\tan(c)$.

E. $\frac{\arctan(c)}{2}$

$$= \int_0^c \frac{dx}{c\left(\left(\frac{x}{\sqrt{c}}\right)^2+1\right)} \underset{u=\frac{x}{\sqrt{c}}}{=} \frac{\sqrt{c}}{c}\int_0^{\sqrt{c}} \frac{du}{u^2+1}$$

$\sqrt{c}\,du = dx$

$$= \frac{\sqrt{c}}{c}\left(\arctan(\sqrt{c}) - \arctan(0)\right)$$

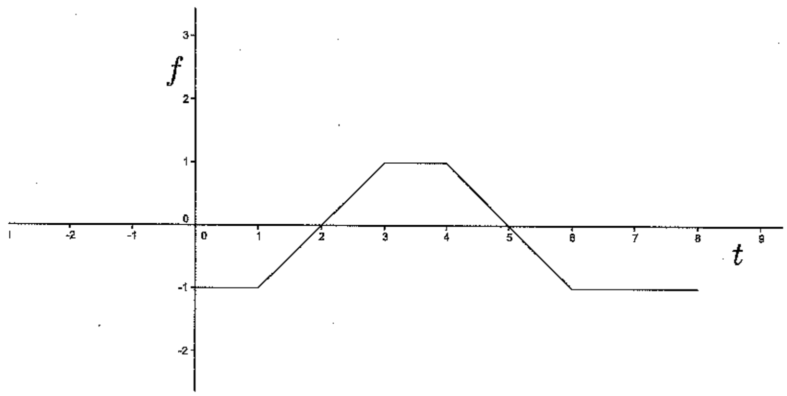
$$= \frac{1}{\sqrt{c}}\arctan(\sqrt{c})$$

Record the correct answer to the following problems on the front page of this exam.

Let

$$F(x) = \int_0^x f(t)dt \quad \text{for } x \text{ in the interval } [0, 8],$$

where $f(t)$ is the function with the graph shown in the following picture. This function F will be used for Problems 7 and 8 on this page.



min max

7. Which of the following statements is true?

A. F has a local maximum at 1 and a local minimum at 6.

B. F has a local maximum at 2 and a local minimum at 5.

C. F has a local minimum at 2 and a local maximum at 5.

D. F has local maxima at 3 and 4.

E. $F(3) = F(4)$.

8. Determine $F(3)$. $= \int_0^3 f(t)\,dt = -1 + 0 = -1$

consider areas

A. -2

B. -1

C. 0

D. 1

E. 2

Record the correct answer to the following problems on the front page of this exam.

9. A particle is traveling along a straight line with a velocity of

$$v(t) = 3t^2 - 6t - 24 \text{ meters/minute.}$$

What is the particle's <u>total</u> distance traveled during the time interval $[1, 5]$?

A. 34 meters

B. 44 meters

C. 54 meters

D. 64 meters

E. 74 meters

$v(t) = 3(t^2 - 2t - 8) = 3(t-4)(t+2) = 0$

for $t = 4$ (and $t = -2$).

$$\int_1^5 |v(t)|\,dt = \int_1^4 -v(t)\,dt + \int_4^5 v(t)\,dt$$

$$= (-t^3 + 3t^2 + 24t)\Big|_1^4 + (t^3 - 3t^2 - 24t)\Big|_4^5$$

$$= (-64 + 48 + 96) - (-1 + 3 + 24)$$

$$+ (125 - 75 - 120) - (64 - 48 - 96)$$

$$= 54 + 10 = \underline{64}$$

10. Determine the integral $\int e^{x+e^x}\,dx$.

$$= \int e^{e^x} \cdot e^x\,dx \underset{\substack{u = e^x \\ du = e^x dx}}{=} \int e^u\,du$$

$$= e^u + C = \underline{e^{e^x} + C}$$

A. $e^{1+e^x} + C$.

B. $(1 + e^x)e^{x+e^x} + C$.

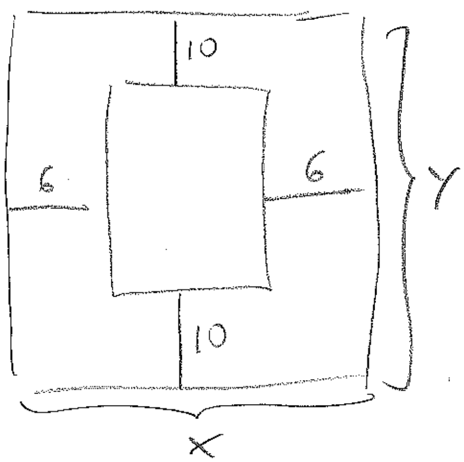
C. $\dfrac{e^{x+e^x}}{1 + e^x} + C$.

D. $e^{e^x} + C$.

E. $e^{x+e^x} + C$.

Free Response Questions: Show your work!

11. A poster of area 6000 cm^2 has blank margins of width 10 centimeters on the top and bottom and 6 centimeters on the sides. Find the total dimensions of the poster that maximize the printed area.



$$xy = 6000$$

$$y = \frac{6000}{x}$$

Printed area

$$A(x) = (x-12)(y-20)$$

$$= (x-12)\left(\frac{6000}{x} - 20\right)$$

So $A(x) = 6000 - \frac{72,000}{x} - 20x + 240$

Maximize A on $(0, \infty)$:

$$A'(x) = \frac{72,000}{x^2} - 20 = 0$$

$$\Leftrightarrow \quad x^2 = 3600$$

$$\Leftrightarrow \quad x = 60$$

	0		60	
$A'(x)$		$+$		$-$
$A(x)$		$\nearrow$		$\searrow$

Hence the local max at $x = 60$ is even an absolute max.

Dimensions are $\boxed{x = 60,\ y = 100}$.

Alternatively with closed interval method: $x \geq 12$, $y \geq 20$ from margins. Hence $\frac{6000}{x} \geq 20$ and $x \leq 300$. Domain of A is $[12, 300]$ and $A(12) = A(300) = 0$.

12. Let $Q(t)$ denote the number of bacteria at time t (measured in hours) in a certain culture. The population grows exponentially, thus

$$Q(t) = Ce^{kt} \text{ for some positive constants } C \text{ and } k.$$

Suppose the population has a doubling time of 20 hours.

(a) Find the constant k. Give the exact answer.

$$k = \frac{\ln(2)}{20}.$$

(b) How long does it take for the population to increase by factor of 10? Give the exact answer and a decimal approximation accurate to two decimal places.

$$e^{kt} = 10$$

$$kt = \ln(10)$$

$$t = \frac{\ln(10)}{k} \underset{(a)}{=} \frac{20 \cdot \ln(10)}{\ln(2)}$$

$$\approx 66.44 \text{ hours}$$

(c) If at time $t = 1$ there are 100 bacteria, how many were there at time $t = 0$? Give the exact answer and a decimal approximation accurate to two decimal places.

$$Ce^{k} = 100$$

$$C = 100e^{-k}$$

$$= 100e^{-\frac{\ln(2)}{20}}$$

$$\approx 96.59 \text{ bacteria}$$

Free Response Questions: Show your work!

13. Determine the following integrals.

(a) $\int_1^2 \left(5x^4 + 8x^3 - 2x + \frac{2}{x^2} - 2\right)\, dx$

$= \left(x^5 + 2x^4 - x^2 - \frac{2}{x} - 2x\right)\Big|_1^2$

$= (32 + 32 - 4 - 1 - 4) - (1 + 2 - 1 - 2 - 2)$

$= \underline{\underline{57}}$

(b) $\int \frac{x}{x+8}\, dx$ $\overline{\overline{\quad}}$ $u = x+8$, $du = dx$ $\quad \int \frac{u-8}{u}\, du = \int 1 - \frac{8}{u}\, du$

$= u - 8 \ln|u| + C = \underline{(x+8) - 8\ln|x+8| + C}$

or $\underline{x - 8\ln|x+8| + C}$

(c) $\int_0^3 \frac{2x}{\sqrt{16+x^2}}\, dx.$ $\overline{\overline{\quad}}$ $u = 16 + x^2$, $du = 2x$ $\quad \int_{16}^{25} \frac{du}{\sqrt{u}}$

$= 2u^{\frac{1}{2}}\Big|_{16}^{25} = 2(\sqrt{25} - \sqrt{16}) = \underline{\underline{2}}$

Free Response Questions: Show your work!

14. (a) Consider the function f on $[2,\infty)$ defined by $f(x) = \int_2^x (u\ln(u))^3\,du$. Explain why the function f is increasing.

$$f'(x) = \underbrace{(\underbrace{x \cdot \ln x}_{>0})^3}_{>0 \text{ on } [2,\infty)}$$

Thus f is increasing on $[2,\infty)$.

(b) Find the derivative of the function g defined by $g(x) = \int_{x^2+1}^{2} (u\ln(u))^3\,du$.

$$g(x) = -f(x^2+1)$$

So

$$g'(x) = -f'(x^2+1)\cdot 2x$$

$$= -2x\left((x^2+1)\cdot \ln(x^2+1)\right)^3$$

Free Response Questions: Show your work!

15. Compute the area of the region enclosed by the graphs of the two functions

$$f(x) = x^2 - x - 2 \text{ and } g(x) = x + 1.$$

Present also a sketch of the two graphs.

$f(x) = g(x)$

$x^2 - x - 2 = x + 1$

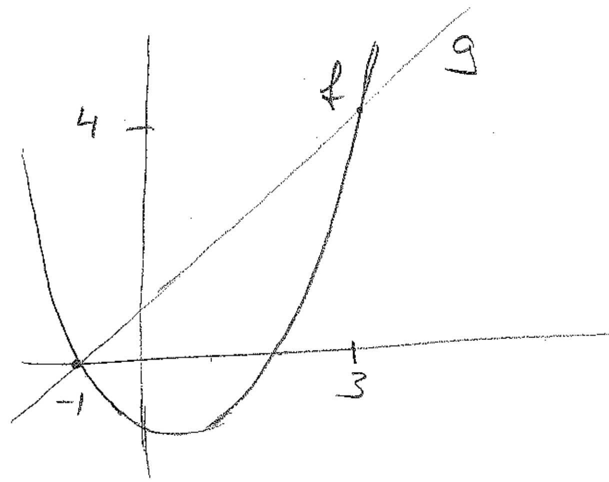
$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, x = 3$

$f(-1) = g(-1) = 0$

$f(3) = g(3) = 4$



$$\int_{-1}^{3} (g-f)(x)\,dx = \int_{-1}^{3} -x^2 + 2x + 3\,dx$$

$$= \left(-\frac{1}{3}x^3 + x^2 + 3x\right)\Big|_{-1}^{3} = -\cancel{9 + 9} + 9 - \left(\frac{1}{3} + 1 - 3\right)$$

$$= \frac{32}{3}$$