MA 113 Calculus I Fall 2013
Exam 4 Wednesday, 18 December 2013

Name: KEY

Section: ______

Last 4 digits of student ID #: ______

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	(D)	E
2	A	B	(C)	D	E
3	A	(B)	C	D	E
4	A	(B)	C	D	E
5	A	B	(C)	D	E
6	A	B	(C)	D	E
7	A	B	C	(D)	E
8	A	B	C	D	(E)
9	A	(B)	C	D	E
10	A	(B)	C	D	E

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Find the linearization, $L(x)$, of the function $f(x) = \frac{3x+1}{2x-1}$ at $x = 3$.

   (A) $L(x) = -5(x-3) + 2$

   (B) $L(x) = 2(x-3) + 2$

   (C) $L(x) = \frac{1}{5}(x-2) + 3$

   (D) $L(x) = -\frac{1}{5}(x-3) + 2$

   (E) $L(x) = \frac{1}{5}(x-3) + 2$

2. Consider the curve given by the equation $y^2 + xe^{2y} = 2$. Which of the following is the slope of the tangent line at the point $(2, 0)$.

   (A) 0

   (B) The curve does not have a tangent line at $(2, 0)$.

   (C) $-\frac{1}{4}$

   (D) $\frac{1}{4}$

   (E) 2

Record the correct answer to the following problems on the front page of this exam.

3. Let $c > 0$. Consider the function $f(x) = 8x^2 + c\ln(x)$ on the interval $(0, \infty)$. Which of the following is a point of inflection of f?

(A) $(\sqrt{c}, f(\sqrt{c}))$

(B) $(\frac{\sqrt{c}}{4}, f(\frac{\sqrt{c}}{4}))$

(C) $(\frac{c}{4}, f(\frac{c}{4}))$

(D) $(\frac{\sqrt{c}}{2}, f(\frac{\sqrt{c}}{2}))$

(E) f does not have a point of inflection.

4. Let $g(x)$ be a differentiable function such that $\lim_{x\to 0} g(x) = 1$ and $\lim_{x\to 0} g'(x) = 4$. Compute the limit

$$\lim_{x\to 0} \frac{x\cos(x)}{g(x) - 1}.$$

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

(E) ∞

Record the correct answer to the following problems on the front page of this exam.

5. Let f be a differentiable function on $\mathbb{R} = (-\infty, \infty)$, and

$$f(2) = 6, \ f'(2) = 5.$$

Let $h(x) = xf(x^2 - 2)$. Compute $h'(2)$.

(A) 26

(B) 36

(C) 46

(D) 56

(E) 66

6. Determine the constant b such that $\int_1^{e^2} \frac{b}{x}\,dx = 8$.

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

Record the correct answer to the following problems on the front page of this exam.

7. A particle is traveling along a straight line with a velocity of

$$v(t) = 3t^2 - 6t \text{ meters/minute.}$$

Compute the particle's total distance traveled over the time interval $[1, 4]$.

(A) 16 meters

(B) 18 meters

(C) 20 meters

(D) 22 meters

(E) 24 meters

8. Let b be a constant. Consider the function $f(x) = e^{x^2-2bx-2}$. Find the maximal open interval on which f is decreasing.

(A) (b, ∞)

(B) $(-b, \infty)$

(C) $(-b, b)$

(D) $(-\infty, -b)$

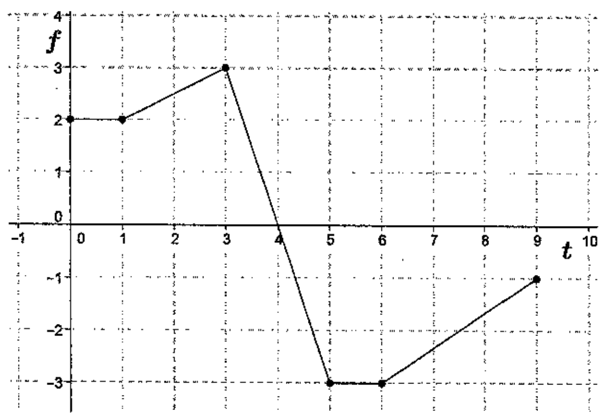
(E) $(-\infty, b)$

Record the correct answer to the following problems on the front page of this exam.

Let

$$F(x) = \int_0^x f(t)dt \quad \text{for } x \text{ in the interval } [0,9],$$

where $f(t)$ is the function with the graph shown in the following picture. This function F will be used for Problems 9 and 10 on this page.



9. Determine $F(3)$.

   (A) 6

   (B) 7

   (C) 8

   (D) 9

   (E) 10

10. Which of the following statements is true?

    (A) F is decreasing on the interval $[3,5]$.

    (B) F has a local maximum at 4.

    (C) F has a local minimum at 4.

    (D) F is increasing on the interval $[6,9]$.

    (E) $F(5) = F(6)$.

Free Response Questions: Show your work!

11. Evaluate the following integrals.

(a) $\displaystyle\int \frac{2e^x}{(5e^x+3)^4}\,dx.$

Let $u = 5e^x + 3$,

$du = 5e^x dx$, so $2e^x dx = \frac{2}{5} du$.) (+1) Identify u; (+1) Compute du

$\int \frac{2e^x}{(5e^x+3)^4} dx = \frac{2}{5}\int \frac{1}{u^4} du$) (+1) Substitute

$= \frac{2}{5}\int u^{-4} du$

$= \frac{2}{5}\cdot\frac{1}{-4+1} u^{-4+1} + C$

$= \frac{2}{5}\cdot\frac{-1}{3} u^{-3} + C$) (+1) Take antiderivative

$= -\frac{2}{15}(5e^x+3)^{-3} + C$) (+1) Substitute, final answer

(b) $\displaystyle\int_1^x \frac{4+t\sin(t)}{2t}\,dt$, where $x > 0$.

$\int_1^x \frac{4+t\sin t}{2t} dt = \int_1^x \left(\frac{2}{t} + \frac{1}{2}\sin t\right) dt$) (+1) Simplify

$= \left(2\ln|t| - \frac{1}{2}\cos t\right)\Big|_1^x$) (+2) Antiderivatives

$= \left[2(\ln(x)) - \frac{1}{2}\cos x\right] - \left[2\ln 1 - \frac{1}{2}\cos 1\right]$) (+1) Evaluate at limits of integration

$= 2\ln x - \frac{1}{2}\cos x + \frac{1}{2}\cos 1$) (+1) Final answer

Free Response Questions: Show your work!

12. Let $B(t)$ denote the number of bacteria at time t (measured in hours) in a certain culture. The population grows exponentially, thus

$$B(t) = Ce^{kt} \text{ for some positive constants } C \text{ and } k.$$

Suppose the population has a doubling time of 20 hours.

(a) Find the constant k. Give the exact answer.

From the textbook, doubling time $= \frac{\ln 2}{k}$, so) (+2) Formula

$k = \frac{\ln 2}{\text{doubling time}} = \frac{\ln 2}{20 \text{ hours}} = \frac{\ln 2}{20}$ hours^{-1}. (+1) Answer (including units)

(b) How long does it take for the population to increase to seven times the initial population? Give the exact answer and a decimal approximation accurate to two decimal places.

Let t = time required for population to increase seven fold.

$7B(0) = 7C = B(t) = Ce^{kt}$.) (+1) Setup

$7\cancel{C} = \cancel{C}e^{kt}$

$\ln 7 = kt$

$t = \frac{\ln 7}{k} = \frac{\ln 7}{\ln 2 / 20 \text{ hours}^{-1}} = 20 \frac{\ln 7}{\ln 2}$ hours) (+1) Exact answer

≈ 56.14 hours.) (+1) decimal approximation (Also accept rounded answer 56.15)

(c) If at time $t = 1$ there are 800 bacteria, how many were there at time $t = 0$? Give the exact answer and a decimal approximation accurate to two decimal places.

$800 = B(1) = Ce^{k \cdot 1} = Ce^{k}$) (+1) Set up

$C = \frac{800}{e^k} = \frac{800}{e^{\ln 2/20}} = \frac{800}{(e^{\ln 2})^{1/20}}$

$= 800/2^{1/20}$ bacteria

$B(0) = Ce^0 = C = \frac{800}{2^{1/20}}$ bacteria

) (+1) Solve for C ($B(0)$) (Exact answer)

≈ 772.74 bacteria) (+1) Decimal approximation (Also accept rounded answer 772.75)

Free Response Questions: Show your work!

13. (a) State both parts of the Fundamental Theorem of Calculus. Use complete sentences and make sure to include all assumptions.

FTOC, Part I: Assume that $f(x)$ is continuous on $[a,b]$. If $F(x)$ is an antiderivative of $f(x)$ on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

(+1) (+1) (+1)

FTOC, Part II: Assume that $f(x)$ is continuous on an open interval I and let $a \in I$. Then the area function

$$A(x) = \int_a^x f(t)\,dt$$

is an antiderivative of $f(x)$ on I; that is, $A'(x) = f(x)$. Equivalently,

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

(+1) (+2)

(b) Find the derivative of the function g defined as $g(x) = \displaystyle\int_2^{x^2-2} t\ln(t^2)\,dt$.

Applying FTOC part II and the Chain Rule yields:

$$\frac{dg}{dx} = \frac{d}{dx}\int_2^{x^2-2} t\ln t^2\,dt$$ (+2) Apply FTOC II

$$= (x^2-2)\cdot\ln(x^2-2)^2\cdot\frac{d}{dx}(x^2-2)$$

$$= (x^2-2)\cdot\ln(x^2-2)^2\cdot 2x$$ (+1) Apply chain rule; compute derivative

$$= 2x(x^2-2)\ln(x^2-2)^2.$$ (+1) Final Answer

Free Response Questions: Show your work!

14. Let u and v be positive real numbers such that $uv = 4$. Find the minimum value of $u^3 + 12v$. Determine the values of u and v for which the minimum is attained. As always, justify your answers!

$uv = 4$, so $v = \frac{4}{u}$.

Substituting gives: $u^3 + 12v = u^3 + 48/u$.) (+1) eliminate v

We seek a minimum value of $f(u) = u^3 + 48u^{-1}$.) (+1) Identify objective function

Since $u, v > 0$, we minimize on the interval $(0, \infty)$.) (+1) identify interval

On an open interval, extreme values occur only at critical points. f is differentiable on $(0,\infty)$, so the only critical points occur where $f'(u) = 0$) (+1) Recognize that we must only check critical points

$f'(u) = 3u^2 - 48u^{-2}$) (+1) derivative

$3u^2 - 48u^{-2} = 0$

$3u^4 - 48 = 0$

$u^4 = 16$

$u = 2$ (since $u > 0$)) (+1) Find critical point

$f'(u)$ is continuous on $(0,\infty)$ and thus can only change sign at $u = 2$. First derivative test:

$f'(1) = 3 - 48 = -45 < 0$, so $f'(u) < 0$ on $(0,2)$

$f'(4) = 3 \cdot 16 - 48/16 = 48 - 3 = 45 > 0$, so $f'(u) > 0$ on $(2,\infty)$.

f' changes from negative to positive at $u = 2$ so f has its minimum at $u = 2$ on the interval $(0,\infty)$.) (+3) Apply first derivative test

Minimum value: $f(2) = 2^3 + \frac{48}{2} = 8 + 24 = 32$) (+1) answer

Free Response Questions: Show your work!

15. Consider the two functions

$$f(x) = x^3 - 7x, \quad g(x) = 2x.$$

(a) Find all points of intersection of the graphs of f and g.

$f(x) = g(x)$

$x^3 - 7x = 2x$) ~~(+1) Set up~~

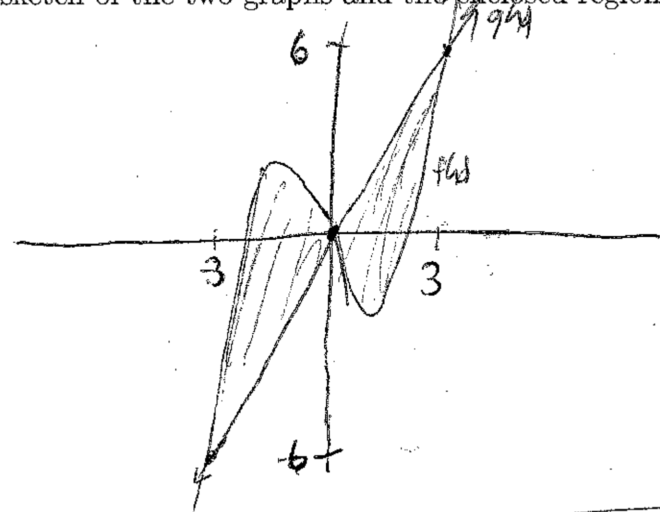
$x^3 - 9x = 0$

$x(x^2 - 9) = x(x-3)(x+3) = 0$) (+1) Factor

The graphs intersect when:

$x = 0$, $x = 3$, and $x = -3$) (+2) Answer

(b) Compute the area of the region enclosed by the graphs of f and g. Present also a sketch of the two graphs and the enclosed region.



(+1) Graph (Rough sketch)

(+1) Formula

$\text{Area} = \int_{-3}^{3} (y_{top} - y_{bottom})\,dx = \int_{-3}^{0} (f(x) - g(x))\,dx + \int_{0}^{3} (g(x) - f(x))\,dx$

$= \int_{-3}^{0} (x^3 - 9x)\,dx + \int_{0}^{3} (9x - x^3)\,dx$

(+2) Split into two integrals; identify y_{top}, y_{bottom}

$= \left(\frac{x^4}{4} - \frac{9}{2}x^2\right)\Big]_{-3}^{0} + \left(\frac{9}{2}x^2 - \frac{1}{4}x^4\right)\Big|_{0}^{3}$] (+1) Antiderivatives

$= \left[0 - \left(\frac{(-3)^4}{4} - \frac{9}{2}(-3)^2\right)\right] + \left[\left(\frac{9}{2}\cdot 3^2 - \frac{1}{4}\cdot 3^4\right) - 0\right]$

$= -\frac{81}{4} + \frac{81}{2} + \frac{81}{2} - \frac{81}{4} = \frac{81}{2}$ (+1) Answer