MA 113 Calculus I Fall 2016
Exam Final Wednesday, December 14, 2016

Name: ____________________

Section: ____________________

Last 4 digits of student ID #: ________

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Free Response Questions: Show your work!

16. A cylindrical can is to hold 20π m^3. The material for the top and bottom costs \$ 10 per m^2 and material for the side costs \$ 8 per m^2. Find the radius r and height h of the least expensive can by carrying out the following steps. Remember that the volume of a cylinder of radius r and height h is $\pi r^2 h$.

(a) Express the total cost of the container in terms of r and h.

$$\text{Cost} = 10 \cdot \pi r^2 + 8 \cdot 2\pi r h.$$

(b) Express the total cost of the container in terms of r alone, and specify the allowed range of r in the problem.

$$\pi r^2 h = 20\pi \Rightarrow h = \frac{20}{r^2}. \quad \text{So,}$$

$$C(r) = \text{cost} = 10\pi r^2 + 16\pi \cdot \frac{20}{r}, \text{ with } r \text{ in } (0, \infty).$$

(c) Find the absolute minimum of the total cost function from part (b). Be sure to use a test that shows that your answer is the *absolute* minimum.

$$C'(r) = 20\pi r - \frac{320\pi}{r^2}.$$ This is defined on $(0,\infty)$, so test $C'(r)=0$

$$0 = 20\pi r - \frac{320\pi}{r^2} \Rightarrow \frac{1}{16} = r^3 \Rightarrow r = \sqrt[3]{16}$$ yields min, by 1^{st} der. test for extreme Values.

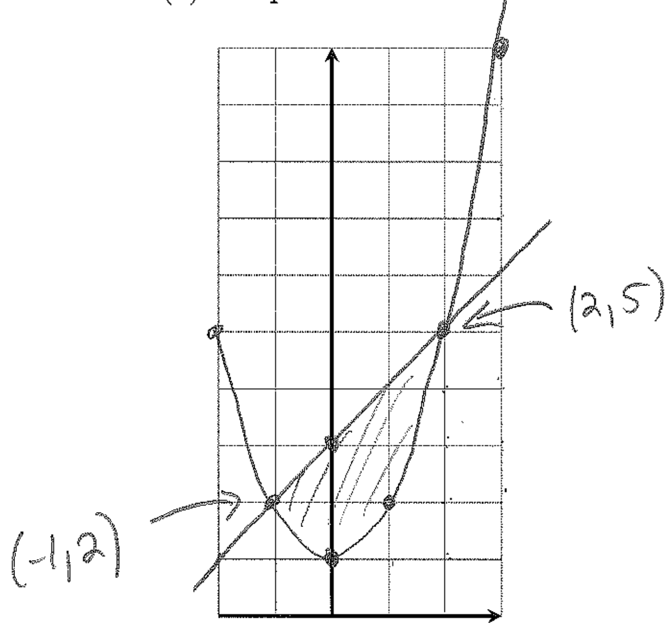
(d) State the values of h and r that achieve the minimum cost.

$$r = \sqrt[3]{16}, \; h = \frac{20}{r^2} = \frac{20}{\sqrt[3]{16^2}}.$$

Free Response Questions: Show your work!

17. The goal of this problem is to find the area between the curves $y = 1+x^2$ and $y = 3+x$.

(a) Graph these functions on the axes provided and identify the points of intersection.



(b) Express the area enclosed between the curves as a definite integral.

$$\int_{-1}^{2} 3+x-(1+x^2)\,dx$$

(c) Evaluate the definite integral.

$$\left[2x+\frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^{2} = \frac{9}{2}$$

Free Response Questions: Show your work!

18. Consider the ellipse given by the equation $x^2/18 + y^2/32 = 1$.

(a) Express dy/dx as a function of x and y.

$$\frac{2x}{18} + \frac{2y}{32} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{y} \cdot \frac{16}{9}.$$

(b) Find the point $(3, y)$ on the ellipse where the y coordinate is positive, that is, $y > 0$.

$$\frac{3^2}{18} + \frac{y^2}{32} = 1 \Rightarrow \frac{y^2}{32} = \frac{1}{2} \Rightarrow y^2 = 16 \Rightarrow \boxed{y = 4, \ (3,4)}$$

(c) Find the equation of the tangent line to the ellipse at the point you found in the previous part. You do not need to simplify your answer.

$$\frac{dy}{dx} = \frac{-3}{4} \cdot \frac{16}{9}.$$

Free Response Questions: Show your work!

19. (a) Use the linear approximation of e^x at $a = 0$ to estimate the value of $\frac{1}{\sqrt{e}}$. Show your work.

$$\underbrace{e^x \approx 1+x}_{} \Rightarrow \frac{1}{\sqrt{e}} = e^{-1/2} \approx 1 - \frac{1}{2} = \frac{1}{2},$$

Since $e^0 = 1$
and $(e^x)' = e^x$,
so $e^0 = 1$ again.

(b) Find the fourth-degree Taylor polynomial at $a = 0$ for e^x. Explain your work.

Set $f(x) = e^x$. Since $f^{(n)}(x) = e^x$, $f^{(n)}(0) = e^0 = 1$ for all n.

Thus, $T_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$.

(c) Use the polynomial you found in part (b) to estimate the value of $\frac{1}{\sqrt{e}}$. Show your work. You do not need to simplify your answer.

$\frac{1}{\sqrt{e}} = e^{-1/2}$, so

$$e^{-1/2} \approx T_4\left(\frac{-1}{2}\right) = 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{8 \cdot 6} + \frac{1}{2^4 \cdot 24}.$$