Name: ________________________________

Section: ______________________________

Last 4 digits of student ID #: ______

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

**On the multiple choice problems:**

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.

- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

**On the free response problems:**

- Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),

- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.
1. Find $\frac{dy}{dx}$ for $y^2 + 3x = x^2y + 1$.

(A) $\frac{3}{2x - 2y}$

(B) $\frac{3}{x^2 - 2y}$

(C) $\frac{3 - 2xy}{2x - 2y}$

(D) $\frac{3 - 2xy}{x^2 - 2y}$

(E) None of the above.

2. A conical tank has height 3 meters and radius 2 meters at the top. Water flows in at a rate of 0.5 $m^3/min$. How fast is the water level rising when it is 0.3 meters? Recall that the volume of a conical tank is $\frac{1}{3}\pi r^2 h$ where $r$ is the radius of the top and $h$ is the height.

(A) $\frac{4.5}{4\pi \cdot 0.09}$ meters per second

(B) $\frac{4.5}{4\pi \cdot 0.3}$ meters per second

(C) $\frac{9}{4\pi \cdot 0.09}$ meters per second

(D) $\frac{9}{4\pi \cdot 0.3}$ meters per second

(E) None of the above.
3. Find the derivative of \( f(x) = x^2 e^{\cos(2x)} \).

(A) \(-2(x \sin(2x) - 1)e^{\cos(2x)}\)
(B) \((x \sin(2x) - 1)e^{\cos(2x)}\)
(C) \(-2(x \sin(2x) - 1)e^{\cos(x)}\)
(D) \(-2x \sin(2x)e^{\cos(2x)}\)
(E) None of the above

4. For an object with position given by some function \( f(x) \), the average velocity of the object on the interval \([a, a + h]\) is given by

\[
\frac{27 - (3 + h)^3}{h}.
\]

Find \( f(x) \) and \( a \).

(A) \( f(x) = x^3, a = 27 \)
(B) \( f(x) = -x^3, a = 3 \)
(C) \( f(x) = -(x + h)^3, a = 3 \)
(D) \( f(x) = -x^3, a = 3^3 \)
(E) None of the above
5. Find the x-coordinate of the point \( P \) on the graph of the function \( y = \sqrt{x} \) closest to the point \((9, 0)\).

(A) 8.5  
(B) 17  
(C) \(\sqrt{8.5}\)  
(D) 9  
(E) None of the above

6. Compute \( L_6 \) to estimate the distance traveled over \([0, 3]\) if the velocity at half-second intervals is as follows:

<table>
<thead>
<tr>
<th>time ( t ) sec</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity ( v(t) ) m/sec</td>
<td>0</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>13</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

(A) 23 meters  
(B) 24.5 meters  
(C) 20.5 meters  
(D) 20 meters  
(E) None of the above
7. Find \( \int 5xe^{3x^2} \, dx \).

\begin{align*}
(A) & \frac{1}{30}e^{3x^2} + C \\
(B) & \frac{5}{6}e^{3x^2} + C \\
(C) & \frac{1}{6}e^{3x^2} + C \\
(D) & e^{3x^2} + C \\
(E) & \text{None of the above}
\end{align*}

8. Use the linear approximation for \( f(x) = \ln(x) \) at \( a = 1 \) to estimate \( \ln(0.89) \).

\begin{align*}
(A) & 0.0000 \\
(B) & -0.8911 \\
(C) & -0.1165 \\
(D) & -0.1100 \\
(E) & \text{None of the above}
\end{align*}
9. Suppose that $f(x)$ is a differentiable function on $(-\infty, \infty)$ with $f(-1) = 2$, and for all $x$ we have $-5 \leq f'(x) \leq 4$. What is the largest possible value for $f(5)$?

(A) 28  
(B) −26  
(C) 26  
(D) −28  
(E) None of the above

10. Find the maximum and minimum values of the function $f(x) = x - \frac{8x}{x + 2}$ on the interval $[0, 3]$.

(A) Min is 0, Max is 2  
(B) Min is $-2$, Max is 0  
(C) Min is $-2$, Max is 2  
(D) Min is $-6$, Max is 2  
(E) None of the above
11. Which, if any, of the following statements is true?

(A) Every function \( f(x) \) with domain \( (-\infty, \infty) \) has a derivative with domain \( (-\infty, \infty) \).
(B) Every continuous function is differentiable.
(C) Every differentiable function with domain \( (-\infty, \infty) \) has a maximum value.
(D) Every continuous function with domain \( (-\infty, \infty) \) has a maximum value.
(E) None of the above

12. Find the value of \( \sum_{i=2}^{6} \sum_{j=2}^{4} (i \cdot j^2) \).

(A) 205
(B) 580
(C) 180
(D) 630
(E) None of the above.
13. Find the following limits. Justify your answers. (Students who guess the answer based on a few values of the function will not receive full credit.)

(a) \( \lim_{x \to \infty} \frac{1 - e^x}{\ln(x + 1)} \)

(b) \( \lim_{x \to \infty} \frac{11x^3 + 9x^2}{2x^3 - 5} \)
14. (a) Find \[ \int (2x^3 + x) \sqrt{x^4 + x^2 + 1} \, dx. \]

(b) Find \[ \int_1^2 \frac{\sin(\ln(10x))}{x} \, dx. \]
15. (a) Find the sixth-degree Taylor polynomial at $a = 0$ for $\cos(x)$. Explain your work.

Recall that $T_N(x) = \sum_{n=0}^{N} \frac{f^{(n)}(a)}{n!} x^n$.

(b) Use the polynomial you found in part (a) to estimate the value of $\cos(3/4)$. Show your work. You do not need to simplify your answer.
16. (a) Compute \( \frac{d}{dx} \int_0^{x^2+1} t^3 + 1 \, dt \).

(b) A particle moves in a straight line with velocity \( 10 - 2t \) meters per second. Find the total distance traveled over the time interval \([0, 8]\). INCLUDE UNITS!!!