Exam 4

Form A

Name: ______________________________ Section and/or TA: ________________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 16 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

SCORE

Multiple Choice	17	18	19	20	Total Score
64	9	9	9	9	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. If $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x) = 1$, then find the value of $\lim_{x\to 2} \dfrac{f(x)+g(x)}{\sqrt{g^2(x)+3}}$.

   A. 1

   B. 2

   C. 3

   D. 4

   E. 5

2. Find the value of $\lim_{x\to\infty} \arcsin\left(\dfrac{x^2-3x+5}{2x^2+x+9}\right)$.

   A. $-\pi/2$

   B. 0

   C. $\pi/6$

   D. $\pi/3$

   E. $\pi/2$

3. Find the values of A and B such that the function $f(x) = \begin{cases} -2x^2+5, & x \leq -1 \\ Ax+B, & -1 < x < 2 \\ 3x^2-3, & 2 \leq x \end{cases}$

   is continuous.

   A. $A = 0,\ B = 9$

   B. $A = 1,\ B = 4$

   C. $A = 1,\ B = 8$

   D. $A = 2,\ B = 5$

   E. $A = 2,\ B = 7$

4. By Intermediate Value Theorem, which interval contains a solution of the equation $2^x + 3 = 3^x$?

   A. $[-1, 0]$

   B. $[0, 1]$

   C. $[1, 2]$

   D. $[2, 3]$

   E. $[3, 4]$

5. Find the derivative of $f(x) = \dfrac{x^2+x}{e^x}$.

   A. $f'(x) = e^{-x}(-x^2+x+1)$.

   B. $f'(x) = \dfrac{2x+1}{e^x}$

   C. $f'(x) = \dfrac{x^2-x-1}{e^x}$

   D. $f'(x) = \dfrac{2x+1}{xe^{x-1}}$

   E. $f'(x) = 2xe^{-x} + x^2e^{-x} + 1$

6. Find the equation of the tangent line to the curve $x^2 + xy - y^2 + 1 = 0$ at the point $(1,2)$.

   A. $y = -10x + 12$

   B. $y = -\dfrac{4}{5}x + \dfrac{14}{5}$

   C. $y = \dfrac{2}{3}x + \dfrac{4}{3}$

   D. $y = \dfrac{4}{3}x + \dfrac{2}{3}$

   E. $y = \dfrac{5}{3}x + \dfrac{1}{3}$

7. Find the derivative of $f(x) = \ln(x^4 + 2x)$, where $x > 0$.

   A. $f'(x) = \dfrac{1}{x^4 + 2x}$

   B. $f'(x) = \dfrac{4x^3 + 2}{x^4 + 2x}$.

   C. $f'(x) = \dfrac{1}{4x^3 + 2}$

   D. $f'(x) = \ln(4x^3 + 2)$

   E. $f'(x) = \dfrac{x^4 + 2x}{4x^3 + 2}$

8. Let $f(x) = |x^2 - 4x + 3|$. Find the point(s) c so that $f'(c)$ does not exist.

   A. $c = 2$

   B. $c = 1$ and $c = 2$

   C. $c = 1$ and $c = 3$

   D. $c = 2$ and $c = 3$

   E. $c = 1$, $c = 2$, and $c = 3$

9. Find a function F which is an anti-derivative of x^3 and satisfies $F(1) = 1$.

    A. $F(x) = 3x^2 - 2$

    B. $F(x) = x^4$

    C. $F(x) = \dfrac{x^3}{3} + \dfrac{2}{3}$

    D. $F(x) = x^3$

    E. $F(x) = \dfrac{x^4}{4} + \dfrac{3}{4}$

10. Suppose that f is a differentiable function on $(0,4)$, that $f'(x) > 0$ for x in each of the intervals $(0,1)$, $(1,2)$ and $(3,4)$ and that $f'(x) < 0$ on the interval $(2,3)$.

    Select the correct statement.

    A. f has a local minimum at 1 and no local maximum.

    B. f has a local minimum at 2 and a local maximum at 3.

    C. f has a local maximum at 2 and a local minimum at 3.

    D. f has local minima at 1 and 3 and a local maximum at 2.

    E. f has local maxima at 1 and 3 and a local minimum at 2.

11. The side length ℓ of a square is increasing. When $\ell = 5$, the derivative $\dfrac{d\ell}{dt} = 6$. Let A be the area of the square and find $\dfrac{dA}{dt}$ when $\ell = 5$.

A. 25

B. 30

C. 50

D. 60

E. 72

12. Use L'Hôpital's rule to find the limit $\lim\limits_{x \to 0} \dfrac{\cos(3x) - 1}{x^2}$.

A. $-9/2$

B. $-3/2$

C. $+3/2$

D. $+3$

E. $+9/2$

13. $\int_0^{\pi/4} \sin^5 x \cos x \, dx = \frac{1}{a}$ where $a =$ ________

A. 16

B. 24

C. 48

D. 49

E. 50

14. $\int_1^5 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx =$

A. $12 + \ln(5)$

B. $16 + \ln(5)$

C. $18 + \ln(5)$

D. $20 + \ln(5)$

E. $22 - \ln(5)$

15. If $f(x) = \int_2^{x^2} \cos\sqrt{t}\,dt$ and $x \geq 0$, find $f'(x)$.

A. $f'(x) = \cos(x)$

B. $f'(x) = 2x\sin(x)$

C. $f'(x) = 2x\cos(x)$.

D. $f'(x) = \sin(x)$

E. $f'(x) = \cos(x) - 2$

16. Find the linearization, $L(x)$, of the function $f(x) = \dfrac{2}{1+e^x}$ at $a = 0$.

A. $L(x) = 1 - \frac{1}{4}x$

B. $L(x) = 1 + \frac{1}{2}x$

C. $L(x) = 1 - \frac{1}{2}x$

D. $L(x) = 1 + x$

E. $L(x) = 1 - x$

Free Response Questions

Show all of your work

17. The tangent line to the graph of a function f at the point $(1, 2)$ is $y = 3x - 1$.
    (a) What is $f(1)$?

    (b) What is $f'(1)$?

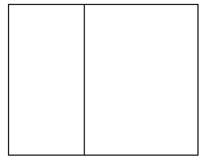
    (c) If $g(x) = f(x^3)$, then find $g'(1)$. Show your work.

18. Find the ***third*** derivatives of the following two functions. Show your work.

    (a) $f(x) = \sin(x) + \cos(x)$

    (b) $g(x) = x^2 \ln(x)$

19. We have 240 meters of fencing and form a rectangular pen that is divided in two by a fence parallel to two of the sides. Find the area and dimensions of the pen which encloses the largest area.



20. A particle is moving along a line with acceleration function $a(t) = 2t + 3$ meters/second^2 and initial velocity $v(0) = -4$ meters/second.

(a) Find the velocity at time t. Show your work.

(b) Find the displacement of the particle over the time interval $0 \leq t \leq 3$. Show your work.

(c) Find the total distance traveled during the time interval $0 \leq t \leq 3$. Show your work.