Name: ________________________________

Section: ________________________________

Last 4 digits of student ID #: _______

This is a two-hour exam. This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.

- Carefully check your answers. No credit will be given for answers other than those indicated on the multiple choice answer box.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit).

- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.
1. Find the slope, the \( y \)-intercept, and the \( x \)-intercept of the line \( 3x + y = 2 \).

(A) slope is \(-3\), \( y \)-intercept is 2, \( x \)-intercept is 2/3
(B) slope is \(-3\), \( y \)-intercept is 2/3, \( x \)-intercept is 3/2
(C) slope is \(-3\), \( y \)-intercept is 2, \( x \)-intercept is 3/2
(D) slope is 3, \( y \)-intercept is 2, \( x \)-intercept is 2/3
(E) None of the above.

2. Let \( f(x) = 2 + \frac{1}{x + 3} \). Determine the inverse function of \( f(x) \).

(A) \( f^{-1}(x) = \frac{1 - 3x}{x} \)
(B) \( f^{-1}(x) = \frac{7 - 3x}{x - 2} \)
(C) \( f^{-1}(x) = \frac{3 - 3x}{x} \)
(D) \( f^{-1}(x) = \frac{1 - 3x}{x - 2} \)
(E) None of the above.

3. Find \( \frac{dy}{dx} \) where \( x^2 + 3xy - y^2 = 2 \).

(A) \( \frac{dy}{dx} = \frac{-2x - 3y}{3x + 2y} \)
(B) \( \frac{dy}{dx} = \frac{2x + 3}{2y} \)
(C) \( \frac{dy}{dx} = \frac{2x}{2y - 3} \)
(D) \( \frac{dy}{dx} = \frac{2x + 3y}{2y - 3x} \)
(E) None of the above
4. Suppose $f(x)$ has domain $(-\infty, \infty)$. If $f'(a)$ does not exist and $f'(x) < 0$ for every $x > a$ and $f'(x) > 0$ for every $x < a$, then which of the following must be true?

(A) $f(x)$ has an absolute maximum at $x = a$
(B) $f(x)$ has an absolute minimum at $x = a$
(C) $f(x)$ is decreasing on $(-\infty, \infty)$
(D) $f(x)$ is increasing on $(-\infty, \infty)$
(E) None of the above

5. The limit $\lim_{h \to 0} \frac{(3 + h)^2 - 9}{h}$ represents a derivative $f'(a)$. Find $f(x)$ and $a$.

(A) $f(x) = 1 + x^2$, $a = 2$
(B) $f(x) = x^2$, $a = 3$
(C) $f(x) = x^2$, $a = 9$
(D) $f(x) = x^2 - 3x$, $a = 3$
(E) None of the above.

6. Compute $R_5$ over the interval $0 \leq t \leq 2$ for the function $v(t) = 1 + 3x$.

(A) 8.6
(B) 6.8
(C) 9.2
(D) 9.5
(E) None of the above

7. Determine the intervals on which $f(x) = x^3 - 2x^2 - x + 2$ is concave up and concave down.

(A) Concave down on $(-\infty, 3/2)$, Concave up on $(3/2, \infty)$
(B) Concave down on $(-\infty, 1/2)$, Concave up on $(1/2, \infty)$
(C) Concave down on $(-\infty, 0)$, Concave up on $(0, \infty)$
(D) Concave up on $(-\infty, 3/2)$, Concave down on $(3/2, \infty)$
(E) None of the above
8. Assume \( x - y = 23. \) Find the \( x \) value where \( (x - 2)y \) is minimized.

(A) \(-23/2\)
(B) \(-21/2\)
(C) \(23/2\)
(D) \(25/2\)
(E) None of the above

9. Let \( A(x) = \int_0^x f(t) \, dt \), with \( f(t) \) given in the graph below. Where does \( A(x) \) have a local maximum on the interval \((0, 3)\)?

(A) \( x = 3/2 \)
(B) \( x = 1 \)
(C) \( x = 3 \)
(D) \( x = 2 \)
(E) None of the above
10. Find the fifth-degree Taylor polynomial approximation of \( \sin x \) centered at \( a = 0 \). Recall that the \( N \)-th degree Taylor polynomial for \( f(x) \) at \( a \) is

\[
T_N(x) = \sum_{n=0}^{N} \frac{f^{(n)}(a)}{n!}(x-a)^n.
\]

(A) \( x - \frac{x^3}{6} + \frac{x^5}{120} \)

(B) \( 1 - \frac{x^2}{2} + \frac{x^4}{24} \)

(C) \( 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} \)

(D) \( 1 + x + x^2 + x^3 + x^4 + x^5 \)

(E) None of the above

11. Suppose that \( g(x) \) is differentiable for all \( x \) and that \( -6 \leq g'(x) \leq 2 \) for all \( x \). If \( g(0) = 3 \), what is the largest possible value of \( g(2) \)?

(A) \( -9 \)

(B) \( 5 \)

(C) \( 7 \)

(D) \( 4 \)

(E) None of the above

12. Find \( \frac{d}{dx} \int_0^{5x^2} e^{-t^2} \, dt \).

(A) \( e^{-x^2} \)

(B) \( 10xe^{-25x^4} \)

(C) \( 5x^2e^{-x^2} \)

(D) \( e^{-25x^4} \)

(E) None of the above.
13. Find the following limits. Justify your answers. (Students who guess the answer based on a few values of the function will not receive full credit.)

(a) \( \lim_{x \to 0} \frac{x^2}{\cos(3x) - 1} \)

(b) \( \lim_{x \to \infty} \frac{9x^8 - x^6 + x - 12}{7x^8 + x^5 - x^2 + 8} \)
14. (a) Find \( \int \frac{1}{x (\ln x)^3} \, dx \).

(b) Find \( \int_{0}^{\pi} x \cos(4x^2) \, dx \).
15. (a) Find the linearization of $f(x) = e^{2x}$ at $a = 0$.

(b) Use the linearization you found in part (a) to estimate the value of $e^{0.2}$. Show your work. No credit will be given for using only a calculator.
16. A particle moves in a straight line so that its velocity at time $t$ seconds is

$$v(t) = t(t - 2)(t - 4) = t^3 - 6t^2 + 8t.$$ 
meters per second.

(a) Find the displacement of the particle during the time interval $1 \leq t \leq 3$. Include units!

(b) Find the total distance traveled by the particle during $1 \leq t \leq 3$. Include units!