Name: ______________________ Section and/or TA: ______________

Do not remove this answer page. You will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or communication capabilities is permitted. You may not use a phone or other communication device during the exam.

The exam consists of 12 multiple choice questions that are worth 5 points each and 4 free response questions that are worth 10 points each. You should work the multiple choice questions on the question page. After you have checked your work carefully, record your answers by completely filling in the circle below that corresponds to your answer. If you must change your answer, make a note on the front of the exam. Be sure to check carefully when you transfer your answers to the cover sheet.

Show all work to receive full credit on the free response problems. You do not need to compute a decimal approximation to your answer. For example, the answer 4π is preferred to 12.57.

Multiple Choice Questions

1	A	B	C	D	E	**7**	A	B	C	D	E
2	A	B	C	D	E	**8**	A	B	C	D	E
3	A	B	C	D	E	**9**	A	B	C	D	E
4	A	B	C	D	E	**10**	A	B	C	D	E
5	A	B	C	D	E	**11**	A	B	C	D	E
6	A	B	C	D	E	**12**	A	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	Total Score
60	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Let

$$f(x) = \begin{cases} 6 - 3cx + x^2 & x < 1 \\ 3 - cx^2 & x \geq 1 \end{cases}$$

For what value(s) of c is this function continuous?

A. $c = 1$

B. $c = -2$

C. $c = 3$

D. $c = 2$

E. There is no value of c for which f is continuous

2. (5 points) Suppose $\cos(x) \leq f(x) \leq 1$. From the possibilities below, which value of a can be used to show that $\lim_{x \to a} f(x)$ exists after applying the squeeze theorem?

A. $a = 2$

B. $a = 0$

C. $a = \pi/2$

D. $a = -1$

E. $a = 1$

3. (5 points) Consider the limit $\lim_{h \to 0} \dfrac{e^{\pi+h} - e^{\pi}}{h} = L$. Select the correct statement.

   A. The value L is the derivative of e^x at $x = 0$.

   B. The value L is the derivative of $e^{\pi+x}$ at $x = \pi$.

   C. The value L is the derivative of $e^{\pi x}$ at $x = 0$.

   D. The value L is the derivative of e^x at $x = \pi$.

   E. The value L is the derivative of e^x at $x = h$.

4. (5 points) Consider the curve defined by $y = y^3 + xy + x^3$. Find the tangent line to this curve at the point $(x, y) = (1, -1)$.

   A. $y + 1 = \frac{2}{3}(x + 1)$

   B. $y - 1 = \frac{1}{2}(x + 1)$

   C. $y - 1 = -\frac{1}{2}(x - 1)$

   D. $y + 1 = -\frac{2}{3}(x - 1)$

   E. $y + 1 = -\frac{1}{4}(x - 1)$

5. (5 points) Suppose that a ball is thrown into the air at time $t = 0$ so that its height above the ground after t seconds is $h(t) = -5t^2 + 12t$ meters. Find the velocity of the ball as it hits the ground.

   A. -48 meters/seconds

   B. 0 meters/seconds

   C. -12 meters/seconds

   D. -1.2 meters/seconds

   E. -24 meters/seconds

6. (5 points) The *derivative* of f is $f'(x) = x(x^2 - 1)$. Find the interval or intervals where f is increasing.

   A. $(-1, 0)$ *and* $(1, \infty)$

   B. $(0, \infty)$

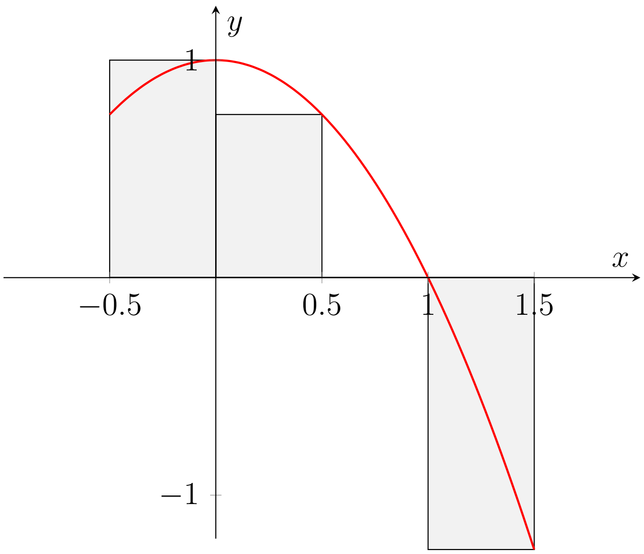
   C. $(-\infty, -1)$ and $(0, 1)$

   D. $(-1, 1)$

   E. $(-\infty, 0)$ and $(1, \infty)$

7. (5 points) Consider the function $f(x) = -x^2+1$ defined on the interval $[-0.5, 1.5]$.

Estimate $\int_{-0.5}^{1.5} (-x^2 + 1)\, dx$ using right endpoints for $n = 4$ approximating rectangles all having bases of the same length, as shown in the picture.



A. 0.25 B. 0.83 C. 0.75 D. 0.5 E. 1.5

8. (5 points) Determine which of the following regions has an area equal to the given limit without evaluating the limit:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \left(5 + \frac{2i}{n}\right)^{10}$$

A. The area of the region under the graph of $y = x^5$ on the interval [2, 5].

B. The area of the region under the graph of $y = (x + 5)^{10}$ on the interval [5, 7].

C. The area of the region under the graph of $y = x^{10}$ on the interval [2, 5].

D. The area of the region under the graph of $y = x^5$ on the interval [5, 7].

E. The area of the region under the graph of $y = x^{10}$ on the interval [5, 7].

9. (5 points) Use L'Hôpital's rule and the Fundamental Theorem of Calculus to evaluate the following limit

$$\lim_{x \to 0} \frac{\displaystyle\int_0^x (5 - 5\cos(t))\, dt}{x^3}$$

A. 1/6

B. 2/3

C. 1/3

D. 0

E. 5/6

10. (5 points) Assuming that $\displaystyle\int_0^5 f(x)\ dx = 5$ and $\displaystyle\int_0^5 g(x)\ dx = 12$, find

$$\int_0^5 \left(2f(x) - \frac{1}{3}g(x)\right)\ dx$$

A. 14

B. 16

C. 7

D. 6

E. 4

11. (5 points) Use the Fundamental Theorem of Calculus to find the function f that verifies the following equation

$$4 + \int_0^x \frac{f(t)}{\sqrt{t}}\, dt = 8x^3$$

A. $f(x) = 24x^2$

B. $f(x) = 24x^{3/2}$

C. $f(x) = 24x^{5/2}$

D. $f(x) = 24x^2 - 4$

E. $f(x) = 2x^4$

12. (5 points) Suppose f is a continuous function such that

$$f(2) = 7 \qquad f'(2) = 12 \qquad f''(2) = 12$$

$$f(4) = 63 \qquad f'(4) = 48 \qquad f''(4) = 24$$

and f' and f'' are continuous everywhere.

Evaluate: $\displaystyle\int_2^4 f'(t)\, dt.$

A. 56

B. 60

C. 70

D. 36

E. 12

Free response questions: Show work clearly with proper notation.

13. (10 points) Compute the derivatives, you do not need to simplify your answers

(a) $\dfrac{d}{dx}(x^2e^{-x^2})$

(b) $\dfrac{d}{dx}\left(\dfrac{\cos(x)}{1-\sin(x)}\right)$

(c) $\dfrac{d}{dt}\sqrt{t^4+1}$

Solution: a) Use the product rule and chain rule to find

$$\frac{d}{dx}(x^2e^{-x^2}) = 2xe^{-x^2} + x^2 \cdot -2xe^{-x^2} = 2xe^{-x^2}(1-x^2)$$

b) Use the quotient rule to write

$$\frac{d}{dx}(\frac{\cos(x)}{1-\sin(x)}) = \frac{-\sin(x)(1-\sin(x)) - (\cos(x))(-\cos(x))}{(1-\sin(x))^2} = \frac{-\sin(x)+1}{(1-\sin(x))^2}.$$

c) We write the radical as a power and use the chain rule to find

$$\frac{d}{dx}(t^4+1)^{1/2} = \frac{1}{2}4t^3(t^4+1)^{-1/2} = \frac{2t^3}{\sqrt{t^4+1}}.$$

Grading:

a) Product rule (2 point), chain rule (1 point), answer (1 point).

b) Quotient rule (2 points), answer (1 point)

c) Derivative of square root (1 point), chain rule (1 point), answer (1 point)

Notes: Problem does not ask for simplification. A student who applies a rule, but with some minor error may receive the credit for the rule, but not for the correct answer.

14. (10 points) Let $f(x) = \int_0^x (t^2 - 16)\, dt$.

(a) Find f'.

(b) Find the intervals where f is increasing and decreasing.

(c) Find the locations of the local maxima or minima, if any.

Solution: WW II-1.3 #10

a) According to FTC I, the derivative of f, $f'(x) = x^2 - 16 = (x-4)(x+4)$. Thus,

- f is increasing in $(-\infty, -4)$ and $(4, \infty)$ and
- decreasing in $(-4, 4)$.

c) There is a local minimum at $x = 4$ and a local maximum at $x = -4$.

Grading:

a) Find f' using FTC I (3 points)

b) Interval of increase and decrease (4 points)

c) Locations of local maxima or minima (3 points)

15. (10 points) Evaluate the following integrals using substitution. You must clearly show steps of the substitution to receive full credit.

 (a) $\displaystyle\int x^2 \cos(x^3)\,dx$

 (b) $\displaystyle\int \frac{1}{x}\ln(x)\,dx$

Solution:

a) Set $u = x^3$, then

$$\int x^2 \cos(x^3)\,dx = \frac{1}{3}\int \cos(u)\,du = \frac{1}{3}\sin(u) + C = \frac{1}{3}\sin(x^3) + C$$

a) Set $u = \ln(u)$, then

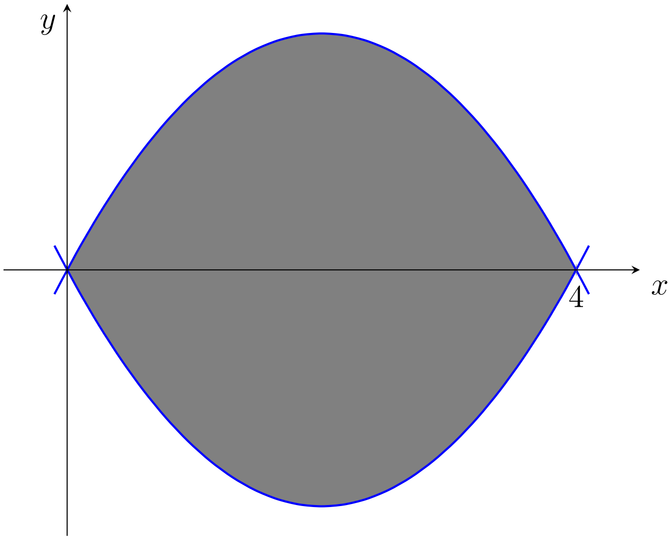
$$\int \frac{1}{x}\ln(x)\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln(\mid x \mid))^2 + C$$

Grading:

a) Set the correct substitution (2 points), simplify and find antiderivative in terms of u (2 point), write answer in terms of x (1 point)

b) Set the correct substitution (2 points), simplify and find antiderivative in terms of u (2 point), write answer in terms of x (1 point)

16. (10 points) Below are the graphs of $f(x) = x^2 - 4x$ and $g(x) = 4x - x^2$.
    (a) Set up an integral whose value is the area of the shaded region.
    (b) Evaluate your integral to find the area of the region.



Solution: WS II-§1.5 #4.

The graphs of $x^2 - 4x$ and $4x - x^2$ intersect for x where $x^2 - 4x = 4x - x^2$ or $2x^2 - 8x = 2x(x-4) = 0$. The solutions are $x = 0, 4$.

The area between the graphs of $y = 4x - x^2$ and $y = x^2 - 4x$ is

$$\int_0^4 [4x - x^2 - (x^2 - 4)]\,dx = \int_0^4 [8x - 2x^2]\,dx.$$

We use FTC II to evaluate the integrals,

$$\begin{aligned}\int_0^4 [8x - 2x^2]\,dx &= \left(\frac{8x^2}{2} - \frac{2x^3}{3}\right)\Big|_{x=0}^{4} \\ &= \left(4^3 - \frac{2(4)^3}{3}\right) - 0 = \frac{64}{3}.\end{aligned}$$

Grading:

Points of intersection (2 points) (may read from graph), expression of area as an integral (3 points), finding anti-derivative (3 points), value for area (2 points).

Students may read points of intersection off of graph. Students who compute the definite integral without computing the antiderivatives should receive half credit.