Worksheet # 4: Basic Limit Laws

1. Given $\lim_{x\to 2} f(x) = 5$ and $\lim_{x\to 2} g(x) = 2$, use limit laws (justify your work) to compute the following limits. Note when working through a limit problem that your answers should be a chain of equalities. Make sure to keep the $\lim_{x\to a}$ operator until the very last step.

   (a) $\lim_{x\to 2} 2f(x) - g(x)$

   (b) $\lim_{x\to 2} \dfrac{f(x)g(x)}{x}$

   (c) $\lim_{x\to 2} f(x)^2 + x \cdot g(x)^2$

   (d) $\lim_{x\to 2} [f(x)]^{\frac{3}{2}}$

2. Calculate the following limits if they exist or explain why the limit does not exist.

   (a) $\lim_{x\to 1} \dfrac{x^2-1}{x-1}$

   (b) $\lim_{x\to 1} \dfrac{x^2-1}{x-2}$

   (c) $\lim_{x\to 2^+} \dfrac{x^2-1}{x-2}$

   (d) $\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$

3. Can the quotient law of limits be applied to evaluate $\lim_{x\to 0} \dfrac{\sin x}{x}$?

4. Find the value of c such that $\lim_{x\to 2} \dfrac{x^2+3x+c}{x-2}$ exists. What is the limit?

5. find the value of c such that $\lim_{x\to 5} \begin{cases} 2x+c & \text{if } x \leq 5 \\ -3x & \text{if } x > 5 \end{cases}$ exists. What is the limit?

6. Show that $\lim_{h\to 0} \dfrac{|h|}{h}$ does not exist by examining one-sided limits. Then sketch the graph of $\dfrac{|h|}{h}$ and check your reasoning.

7. True or False:

   (a) Let $f(x) = \dfrac{(x+2)(x-1)}{x-1}$ and $g(x) = x+2$. Then $f(x) = g(x)$.

   (b) Let $f(x) = \dfrac{(x+2)(x-1)}{x-1}$ and $g(x) = x+2$. Then $\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x)$.

   (c) If both the one-sided limits of $f(x)$ exist as x approaches a, then $\lim_{x\to a} f(x)$ exists.

   (d) If $\lim_{x\to a} f(x)$ exists then $\lim_{x\to a} f(x) = f(a)$.

8. Draw a graph of two functions $f(x)$ and $g(x)$ such that $\lim_{x\to 0}(f(x)+g(x))$ exist but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exist.