

Worksheet # 5: Continuity

1. Comprehension check:

- Define what it means for $f(x)$ to be continuous at the point $x = a$. What does it mean if $f(x)$ is continuous on the interval $[a, b]$? What does it mean to say $f(x)$ is continuous?
- There are three distinct ways in which a function will fail to be continuous at a point $x = a$. Describe the three types of discontinuity. Provide a sketch and an example of each type.
- True or false? Every function is continuous on its domain.
- True or false? The sum, difference, and product of continuous functions are all continuous.
- If $f(x)$ is continuous at $x = a$, what can you say about $\lim_{x \rightarrow a^+} f(x)$?
- Suppose $f(x), g(x)$ are continuous everywhere. What is $\lim_{x \rightarrow a} \frac{f(x)g(x) - f(x)^3}{g(x)^2 + 1}$?

2. Using the definition of continuity and properties of limits, show that the following functions are continuous at the given point a .

- $f(x) = \pi, a = 1$
- $f(x) = \frac{x^2 + 3x + 1}{x + 3}, a = -1$
- $f(x) = \sqrt{x^2 - 9}, a = 4$

3. Give the largest domain on which the following functions are continuous. Use interval notation.

- $f(x) = \frac{x + 1}{x^2 + 4x + 3}$
- $f(x) = \frac{x}{x^2 + 1}$
- $f(x) = \sqrt{2x - 3} + x^2$
- $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ x + 1 & \text{if } 0 < x < 2 \\ -(x - 2)^2 & \text{if } x \geq 2 \end{cases}$

4. Let c be a number and consider the function $f(x) = \begin{cases} cx^2 - 5 & \text{if } x < 1 \\ 10 & \text{if } x = 1 \\ \frac{1}{x} - 2c & \text{if } x > 1 \end{cases}$.

- Find all numbers c such that $\lim_{x \rightarrow 1} f(x)$ exists.
- Is there a number c such that $f(x)$ is continuous at $x = 1$? Justify your answer.

5. Find parameters a and b so that $f(x) = \begin{cases} 2x^2 + 3x & \text{if } x \leq -4 \\ ax + b & \text{if } -4 < x < 3 \\ -x^3 + 4x^2 - 5 & \text{if } 3 \leq x \end{cases}$ is continuous.

6. Suppose that $f(x)$ and $g(x)$ are continuous functions where $f(2) = 5$ and $g(6) = 1$. Compute the following:

- $\lim_{x \rightarrow 2} \frac{[f(x)]^2 + x}{3x + 2}$.
- $\lim_{x \rightarrow 6} \frac{g(x) + 4x}{f\left(\frac{x}{3}\right) - g(x)}$

7. Suppose that: $f(x) = \begin{cases} \frac{x-6}{|x-6|} & \text{for } x \neq 6, \\ 1 & \text{for } x = 6 \end{cases}$

Determine the points at which the function $f(x)$ is discontinuous and state the type of discontinuity.