Worksheet # 17: Linear Approximation and Applications

1. For each of the following, use a linear approximation to the change in the function and a convenient nearby point to estimate the value:
   (a) \((3.01)^3\)
   (b) \(\sqrt{17}\)
   (c) \(8.06^{2/3}\)
   (d) \(\tan(44^\circ)\)

2. What is the relation between the linearization of a function \(f(x)\) at \(x = a\) and the tangent line to the graph of the function \(f(x)\) at \(x = a\) on the graph?

3. Use the linearization of \(\sqrt{x}\) at \(x = 16\) to estimate \(\sqrt{18}\):
   (a) Find a decimal approximation to \(\sqrt{18}\) using a calculator.
   (b) Compute both the error and the percentage error.

4. Suppose we want to paint a sphere of radius 200 cm with a coat of paint .2 cm thick. Use a linear approximation to approximate the amount of paint we need to do the job.

5. Let \(f(x) = \sqrt{16 + x}\). First, find the linearization to \(f(x)\) at \(x = 0\), then use the linearization to estimate \(\sqrt{15.75}\). Present your solution as a rational number.

6. Find the linearization \(L(x)\) to the function \(f(x) = \sqrt{1 - 2x}\) at \(x = -4\).

7. Find the linearization \(L(x)\) to the function \(f(x) = \sqrt{x + 4}\) at \(x = 4\), then use the linearization to estimate \(\sqrt{8.25}\).

8. Your physics professor tells you that you can replace \(\sin(\theta)\) with \(\theta\) when \(\theta\) is close to zero. Explain why this is reasonable.

9. Suppose we measure the radius of a sphere as 10 cm with an accuracy of \(\pm .5\) cm. Use linear approximations to estimate the maximum error in:
   (a) the computed surface area.
   (b) the computed volume.

10. Suppose that \(y = y(x)\) is a differentiable function which is defined near \(x = 2\), satisfies \(y(2) = -1\) and \(x^2 + 3xy^2 + y^3 = 9\).
    Use the linear approximation to the change in \(y\) to approximate the value of \(y(1.91)\).