Worksheet # 23.5: Review for Exam III

1. (a) Find the linear approximation, $L(x)$, to $f(x) = \sqrt{1-2x}$ at $x = -4$.
   (b) Use the result of (a) to approximate $\sqrt{11}$.
   (c) Find the absolute error in the approximation of $\sqrt{11}$ by using your calculator.

2. (a) Describe in words and diagrams how to use the first and second derivative tests to identify and classify extrema of a function $f(x)$.
   (b) Use the first derivative test to identify and classify the extrema of the function
   $$f(x) = 2x^3 + 3x^2 - 72x - 47.$$

3. Find the absolute minimum of the function $f(t) = t + \sqrt{1-t^2}$ on the interval $[-1, 1]$. Be sure to specify the value of t where the minimum is attained.

4. For each of the following functions (i) Find the intervals on which f is increasing or decreasing. (ii) Find the local maximum and minimum values of f. (iii) Find the intervals of concavity and the inflection points.
   (a) $f(x) = x^4 - 2x^2 + 3$
   (b) $f(x) = e^{2x} + e^{-x}$

5. For what values of c does the polynomial $p(x) = x^4 + cx^3 + x^2$ have two inflection points? One inflection point? No inflection points?

6. (a) State the Mean Value Theorem. Use complete sentences.
   (b) Does there exist a function f such that $f(0) = -1, f(2) = 4$, and $f'(x) \leq 2$ for all x?

7. (a) State L'Hospital's Rule for limits in indeterminate form of type $0/0$. Use complete sentences, and include all necessary assumptions.
   (b) Evaluate $\displaystyle\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$
   (c) Evaluate $\displaystyle\lim_{x \to 0^+} x^3 \ln(x)$
   (d) Evaluate $\displaystyle\lim_{x \to -\infty} \frac{x+2}{\sqrt{9x^2+1}}$
   (e) Evaluate $\displaystyle\lim_{x \to 2} \frac{e^{2x}}{x+2}$

8. A poster is to have an area of 180 cm^2 with 1 cm margins at the bottom and sides and 2 cm margins at the top. What dimensions will give the largest printed area. Be sure to explain how you know you have found the largest area.
   (a) Draw a picture and write the constraint equation.
   (b) Write the function you are asked to maximize or minimize and determine its domain.
   (c) Find the maximum or minimum of the function that you found in part (c).

9. Find a positive number such that the sum of the number and twice its reciprocal is small as possible.

10. Let $f(x) = x^2 - 3x + 1$, $x_1 = 3$. Apply Newton's Method to $f(x)$ and initial guess x_1 to calculate x_2, x_3, x_4.

11. Find the most general anti-derivative of $f(x) = x^2 + \cos(2x + 1)$.

12. Find a function with $f''(x) = \sin(2x)$, $f(\pi) = 1$, and $f(0) = 2$.

13. Find the left endpoint approximation with 3 subdivisions to the area of the region under the graph of $f(x) = 1/x$ for $1 \leq x \leq 2$.

14. We know $\sum_{k=1}^{n} k = n(n+1)/2$ for $n = 1, 2, \ldots$. Find $\sum_{k=5}^{20}(4k+1)$.