Exam 3 Review Sheet

Important Notice

Be sure that you know how to state and use the following theorems for the third exam:

- Theorem 1, page 216, existence of extrema on a closed interval
- Theorem 1, page 226, the Mean Value Theorem

Remember that any good theorem consists of an *hypothesis* and a *conclusion*, and that you must state each completely to receive full credit.

One of these theorems <u>will</u> appear on Exam 3!

Graphing, Extreme Values (Sections 2.7, 4.2-4.4 and Section 4.6

- Know what the *absolute maximum* and *absolute* maximum of a function on a closed interval are, and know that a *local maximum* and *local minimum* are.
- Know that a *critical point* of a function $f(x)$ is a value c for which $f'(c) = 0$ or $f'(c)$ does not exist
- Know *Fermat's Theorem* which states that if $f(c)$ is a local minimum or a local maximum of $f(x)$, then c is a critical point of f. Know that the converse is false (a point c can be a critical point but $f(c)$ is neither a local maximum nor a local minimum. Consider $f(x) = x^3$ and $c = 0$.)
- Know that if f is continuous on $[a, b]$ and differetiable on (a, b), then the minimm or maximum values of f occur either at critical points or at the endpoints a and b.
- Know the *Mean Value Theorem* and its relationship to *Rolle's Theorem*.
- Know the following consequences of the Mean Value Theorem:
  - If $f'(x) = 0$ on an open interval I, then $f(x)$ is constant on I

- If $f'(x) > 0$ for all x in an open interval I, then $f(x)$ is increasing on I
- If $f'(x) < 0$ for all x in an open interval I, then $f(x)$ is decreasing on I
- *First Derivative Test*: If c is a critial point, and $f'(x)$ changes sign from $+$ to $-$ at c, then $f(c)$ is a local maximum. If $f'(x)$ changes sign from $-$ to $+$ at c, then $f(c)$ is a local minimum.

- Know the *Second Derivative Test* for local maxima and local minima: if c is a critical point, then

  - if $f''(c) < 0$, then $f(c)$ is a local maximum
  - if $f''(c) > 0$, then $f(c)$ is a local minimum
  - if $f''(c) = 0$, then the second derivative test fails

  Remember three basic examples for the second derivative test:

  - If $f(x) = -x^2$, then f has a local (in fact global) maximum at 0, and $f''(0) = -2$
  - If $f(x) = x^2$, then f has a local (in fact global) minimum at 0, and $f''(0) = +2$
  - If $f(x) = x^3$, then f has neither a local maximum nor a local minimum at 0, and $f''(0) = 0$.

- Know that if $f''(x) > 0$ on an open interval I, then the graph of f is *concave up* on I, while if $f''(x) < 0$ on I, then the graph of f is *concave down* on I. Know that a point c where f'' changes sign is called a *point of inflection.*

- Know that, to graph a function using Calculus, you need to:

  - Find $f'(x)$, find its critical points, and make a sign graph to determine intervals of increase and decrease and find local maxima and minima
  - Find $f''(x)$, make a sign graph, and find intervals where f is concave up or concave down together with inflection points

- If graphing f on an infinite interval, find any horizontal asymptotes by determining limits at $\pm\infty$. Remember the techniques for finding limits at infinity from Section 2.7, especially Theorem 2 on page 102
- Evaluate f at any critical points or points of inflection, and determine x and y intercepts

Applied Optimization

- Know that to optimize means to find the "best," "most efficient," or "least costly" way of doing something. Know that, to solve an optimization, you need to find an *objective function* to be maximized or minimized and express it as a function of one variable. You'll then apply what you've learned about finding maxima and minima of functions to *optimize*.
- Know the three-step method for solving optimization problems:

Step 1 *Choose variables.* Determine which quantities are relevant, often by drawing a diagram, and assign appropriate variables

Step 2 *Find the objective function and its interval of definition.* Restate the original problem as an optimization problem for a function f over an interval. If f depends on more than one variable, use a *constraint equation* to write f as a function of only one variable.

Step 3 *Optimize the objective function.*

- Know that if the objective function is defined on a closed interval, you must check the endpoints. If the objective function is defined on an open interval, any maximum or minimum must occur at a critical point. To determine whether a maximum or minimum exists, you must analyze the behavior of f as it approaches the endpoints of the interval.

L'Hospital's Rule (Section 4.5)

- Know what an *indeterminate form* is (limits of the form $0/0$, ∞/∞, $0 \cdot \infty$, etc.–see the explanations and examples in section 4.5).
- Know L'Hospital's Rule, (Theorem 1 page 241) and L'Hospital's Rule for limits at infinity (Theorem 2 page 244)

- Know that L'Hospital's rule works for left- or right-hand limits

Linear Approximation and Newton's Method (Sections 4.1, 4.8

Both section 4.1 and 4.8 use the idea of approximating a function near a point $(a, f(a))$ by the function whose graph is the tangent to the graph of $f(x)$ at $x = a$. That function is called the *linearization* of $f(x)$ and is given by

$$L(x) = f(a) + f'(a)(x - a)$$

- Know that, if Δf is the change in $f(x)$ due to a change Δx in its input, then the exact change is

  $$\Delta f = f(a + \Delta x) - f(a)$$

  and the *linear approximation* to Δf is

  $$\Delta f \simeq f'(a)\Delta x.$$

- Know how to estimate small changes in the value of a function by linear approximation

- Know what *absolute error* and *percentage error* in an approximation are

- Know that *Newton's method* is a means of computing solutions to the equation $f(x) = 0$ when f is a differentiable function. Given an first guess x_0, Newton's method generates a series of approximations $x_1, x_2, x_3, \cdots$ by the formula

  $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

  Understand Newton's method graphically (see, for example, the graphs on page 269).

Antiderivatives (Section 4.9)

- Know what an *antiderivative* of a function $f(x)$ is. Know that the *general antiderivative* of a function $f(x)$ takes the form $F(x)+C$ where $F'(x) = f(x)$ and C is a constant

- Know the basic rules for antidifferentiation: the power rule, the sum rule, the multiples rule, and the formulas for antiderivatives of "building block" functions like powers, trig functions, exponentials.

- Know that an antiderivative can be thought of as a solution to a *differential equation*

  $$\frac{dy}{dx} = f(x)$$

  and know what an *initial value problem* is.