Quiz 2 — SEPTEMBER 12TH, 2013 (THURSDAY) MA 113, FALL 2013

Name: KEY Section: ____ total pts: /10

1. (a) Explain why we cannot use the limit law for the limit of a quotient to write

$$\lim_{x\to 3} \frac{x^2+x-12}{x-3} = \frac{\lim_{x\to 3} x^2+x-12}{\lim_{x\to 3} x-3}.$$

(b) Simplify the expression $\frac{x^2+x-12}{x-3}$ so that it is easy to apply the limit laws and evaluate the limit $\lim_{x\to 3} \frac{x^2+x-12}{x-3}$.

a) The quotient limit law requires that the limit of the denominator does not equal zero, but $\lim_{x\to 3}(x-3) = 0$. Therefore, we cannot apply it.

b) $$\lim_{x\to 3} \frac{x^2+x-12}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+4)}{(x-3)} = \lim_{x\to 3}(x+4) = 7$$

2. Let f be defined by

$$f(x) = \begin{cases} x^2 + kx & \text{if } x \le 2 \\ x - 4 & \text{if } x > 2 \end{cases}$$

where k is a constant.

(a) Give the value of the one-sided limits $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$.

(b) Find the constant k which makes f continuous. Explain how you found k.

a) $$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x^2+kx) = (2)^2 + k(2) = 4 + 2k$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x-4) = 2 - 4 = -2$$

b) f is continuous when $x \neq 2$ already so we just need to verify continuity at 2. f is continuous at $x=2$ if

$$f(2) = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

Hence $-2 = 4 + 2k = -2$

so $2k = -6$

and $k = -3$ makes f continuous everywhere