MA 113 Name ____________________

Quiz 8 November 6th, 2013

1. Use L'Hôpital's Rule to evaluate the limit, or state that L'Hôpital's Rule does not apply and explain why it does not. (2 points each)

   (a)
   $$\lim_{x\to 0} \frac{\sin(\sin(\sin(x)))}{x^2-1}$$

   (b)
   $$\lim_{x\to -1} \frac{9x^7+2x^2+7}{x^2-1}$$

Solutions:

(a)
$$\lim_{x\to 0} \frac{\sin(\sin(\sin(x)))}{x^2-1} = \frac{0}{-1} = 0$$

L'Hôpital's Rule does not apply here, as we do not have an indeterminate form. This limit can simply be found using direct substitution.

(b)
$$\frac{9(-1)^7+2(-1)^2+7}{(-1)^2-1} = \frac{0}{0}$$

Direct substitution of -1 yields the indeterminate form $0/0$, so we can apply L'Hôpital's Rule. Thus,

$$\lim_{x\to -1} \frac{9x^7+2x^2+7}{x^2+1} = \lim_{x\to -1} \frac{63x^6+4x}{2x} = \frac{63(-1)^6+4(-1)}{2(-1)} = -\frac{59}{2}$$

2. A box with no top is to be constructed from a single rectangular piece of cardboard, with side lengths A and B. The box will be constructed by cutting out squares of length x from each corner and folding up the sides.

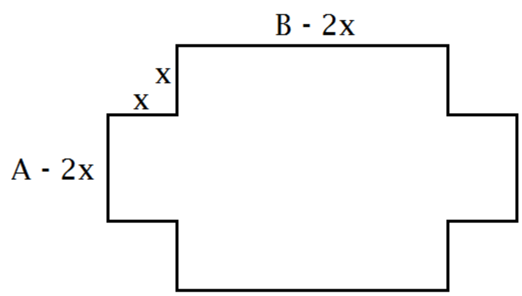
   (a) Draw a picture to describe the situation. Label all pertinent information. (1 point)

   (b) Find the value of x that maximizes the volume of the box if $A = 15$ and $B = 24$. (4 points)

   (c) What are the dimensions of the box that maximizes volume? (1 point)

Solutions:

(a) One possible drawing:



(b) The volume equation for the box is given by $V = x(15 - 2x)(24 - 2x)$. To avoid using the product rule, this can be expanded to $V = 4x^3 - 78x^2 + 360x$. Since we are working with a physical box, we will only consider positive side lengths. Thus, we will be maximizing the volume function on the interval $[0, 7.5]$.

To obtain critical values, we begin by differentiating our formula, which yields $\frac{dV}{dx} = 12x^2 - 156x + 360$. We then set this equal to zero and solve for x:

$$\begin{aligned}
12x^2 - 156x + 360 &= 0 \\
x^2 - 13x + 30 &= 0 \\
(x - 10)(x - 3) &= 0 \\
x &= 3, 10
\end{aligned}$$

The solution of $x = 10$ is not in our interval. Thus the critical values are 0, 3, and 7.5. The critical values of 0 and 7.5 are those which give the minimum volume of 0. The value $x = 3$ yields the desired maximum $V = 486$.

(c) Substituting $x = 3$ from (b) into (x), $(15 - 2x)$, and $(24 - 2x)$ reveals the dimensions of this box to be $3 \times 9 \times 18$