Quiz # 1 — 09/4/14

Answer all questions in a clear and concise manner. Remember that answers without explanation or that are poorly presented may not receive full credit.

1. Which of the following functions have an inverse whose domain is the whole real line? For each function that has such an inverse, calculate that inverse explicitly.

a. $f(x) = x^2$

b. $f(x) = (x+2)^3$

c. $f(x) = \frac{1}{x-1}$

Solution:

a. This function is not one-to-one since $f(-x) = x^2 = f(x)$ for every x. Therefore it cannot have an inverse defined on the whole real line.

b. This function is one-to-one. We calculate the inverse in the usual way beginning with

$$y = (x+2)^3$$

and switching the $x's$ and $y's$ to arrive at

$$x = (y+2)^3.$$

From here, we simply need to solve for y as follows:

$$x = (y+2)^3 \implies x^{\frac{1}{3}} = y+2$$
$$\implies y = -2 + x^{\frac{1}{3}}.$$

c. Note that $\frac{1}{x-1} \neq 0$ for any x in $\mathbb{R}$. Thus $x = 0$ cannot be in the domain of the inverse.

2. Find all the values of x in the interval $[0, \frac{\pi}{2}]$ that satisfy the equation

$$\sin(2x) = \tan(x).$$

Solution: $\sin(2x) = \tan(x) \Rightarrow 2\sin(x)\cos(x) = \frac{\sin(x)}{\cos(x)} \Rightarrow \sin(x)[2\cos(x) - \frac{1}{\cos(x)}] = 0$

Therefore, either $\sin(x) = 0$ and $x = 0$.

Or $[2\cos(x) - \frac{1}{\cos(x)}] = 0 \Rightarrow \cos^2(x) = 1/2 \Rightarrow \cos(x) = \frac{\sqrt{2}}{2}$ (the negative root gives an angle outside $[0, \frac{\pi}{2}]$), and $x = \frac{\pi}{4}$.