Quiz # 3 — 09/18/14

Answer all questions in a clear and concise manner. Remember that answers without explanation or that are poorly presented may not receive full credit.

1. (a) State what it means for a function $f(x)$ to be continuous at a point c.

   (b) Consider the function

$$f(x) = \begin{cases} x^2 + 3 & \text{for } x < 1 \\ 10 - x & \text{for } 1 \leq x \leq 2 \\ 6x - x^2 & \text{for } x > 2. \end{cases}$$

   i. Is the function continuous at $x = 1$?

   ii. Is the function continuous at $x = 2$?

Solution: (a) A function $f(x)$ is continuous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$.

(b) (i) This function is not continuous at $x = 1$ because

$$\lim_{x \to 1-} x^2 + 3 = 4 \qquad \text{and} \qquad \lim_{x \to 1^+} 10 - x = 9$$

so the limit $\lim_{x \to 1} f(x)$ does not exist.

(ii) This function is continuous at $x = 2$. First,

$$\lim_{x \to 2^-} 10 - x = 8 \qquad \text{and} \qquad \lim_{x \to 2^+} 6x - x^2 = 8$$

so $\lim_{x \to 2} f(x) = 8$. Also, $f(2) = 8$. So $\lim_{x \to 2} f(x) = f(2)$.

2. Use the basic limit laws to evaluate the following limit. Be sure to state which limit law you use in each step.

$$\lim_{x\to 2} \frac{x^2+2x+1}{x+5}$$

Solution:

$$\begin{aligned}
\lim_{x\to 2} \frac{x^2+2x+1}{x+5} &= \frac{\lim_{x\to 2}(x^2+2x+1)}{\lim_{x\to 2}(x+5)} && \text{Quotient Law} \\
&= \frac{\lim_{x\to 2} x^2 + \lim_{x\to 2} 2x + \lim_{x\to 2} 1}{\lim_{x\to 2} x + \lim_{x\to 2} 5} && \text{Sum Law} \\
&= \frac{(\lim_{x\to 2} x)^2 + 2\lim_{x\to 2} x + \lim_{x\to 2} 1}{\lim_{x\to 2} x + \lim_{x\to 2} 5} && \text{Constant Multiple \& Power Laws} \\
&= \frac{(2)^2+2(2)+1}{(2)+5} = \frac{9}{7}
\end{aligned}$$