Quiz # 7— 11/6/14

Answer all questions in a clear and concise manner. Remember that answers without explanation or that are poorly presented may not receive full credit.

1. Calculate the limit of

$$\lim_{x\to 0}\frac{\sin(2x)}{\sin(7x)}$$

Solution.

$$\begin{aligned}
&= \lim_{x\to 0}\frac{(\sin(2x))'}{(\sin(7x))'} \text{ by L'Hôpital's Rule,}\\
&= \lim_{x\to 0}\frac{2\cos(2x)}{7\cos(7x)}\\
&= \boxed{\frac{2}{7}}
\end{aligned}$$

2. Describe the local maxima, local minima, and intervals of concavity of

$$f(x) = x + \frac{1}{x}.$$

Solution.

$$f'(x) = 1 - \frac{1}{x^2}$$

Critical points at $x = \pm 1$. Note that $x = 0$ is not in the domain of $f(x)$.

$$f''(x) = \frac{2}{x^3}$$

For $x < 0$, $f''(x) < 0$ and so $f(x)$ is concave down. For $x > 0$, $f''(x) > 0$ and so $f(x)$ is concave up.
It follows that $x = -1$ is a local maximum and $x = 1$ is a local minimum.

3. Assume that $g(x)$ is a differentiable function and that $-1 \le g'(x) \le 2$ for all values of x. Use the mean value theorem to find the largest possible value for $g(3)$ if $g(0) = 1$.

Solution. For all $x \in (0, 3)$ we have $g'(x) \le 2$. The Mean Value Theorem implies that for some $c \in (0, 3)$ we have

$$\frac{g(3) - g(0)}{3 - 0} = g'(c) \le 2.$$

This gives $g(3) = g(0) + 3g'(c) \le 1 + 3\cdot 2 = 7$, and so $g(3) \le 7$. The example $g(x) = 2x + 1$ shows that $g(3) = 7$ can occur.