Quiz #9 for MA 113 - Calculus I (solution)

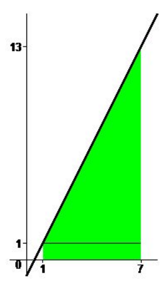
December 4, 2014

1. Consider the function $f(x) = 2x - 1$ on the interval $[1, 7]$.
(a) Compute the Riemann sum for f on the interval $[1, 7]$ with $n = 3$ subintervals and the right endpoints as sample points.
(b) Use a geometric argument to compute the area under the curve $f(x)$ on the interval $[1, 7]$.
(c) Use the Fundamental Theorem of Calculus to evaluate the definite integral
$\int_1^7 (2x - 1)dx$

Solution:
(a) $\Delta_3 = \frac{7-1}{3} = 2$. The right end points are $\{1 + \Delta_3, 1 + 2\Delta_3, 1 + 3\Delta_3\}$ so the Riemann sum is

$f(1 + \Delta_3)\Delta_3 + f(1 + 2\Delta_3)\Delta_3 + f(1 + 3\Delta_3)\Delta_3 =$
$(2 \cdot (1 + \Delta_3) - 1)\Delta_3 + (2 \cdot (1 + 2\Delta_3) - 1)\Delta_3 + (2 \cdot (1 + 3\Delta_3) - 1)\Delta_3 =$
$(2 \cdot (1 + 2) - 1) \cdot 2 + (2 \cdot (1 + 2 \cdot 2) - 1) \cdot 2 + (2 \cdot (1 + 3 \cdot 2) - 1)) \cdot 2 = 10 + 18 + 26 = 54$

(b) Geometrically, $\int_1^7 f(x)dx$ is the area of the figure comprised of the right triangle with vertices $(1, 1), (7, 13)$ and $(7, 1)$ and the rectangle with vertices $(1, 0), (7, 0), (7, 1)$ and $(1, 1)$. The triangle has height 12 and base 6 so its area is 36 while the rectangle has area 6. Thus the total area and hence $\int_1^7 f(x)dx$ is $36 + 6 = 42$.



(c) $f(x)$ is continuous on $[1, 7]$ so the fundamental theorem applies. $\int (2x-1)dx = x^2-x+C$ so $\int_1^7 (2x - 1)dx = x^2 - x \,|_1^7 = 42$.

2. Find the critical numbers of $g(x) = \int_0^x e^{t^2}(t^2 - 5t + 6)\, t\, dt$.

Solution: By the fundamental theorem, $g'(x) = e^{x^2}(x^2 - 5x + 6)\, x = e^{x^2}(x - 3)(x - 2)x$, which is defined everywhere. Thus the only critical numbers are the roots of g' which are $\{0, 2, 3\}$.