# Quiz # 8 — 11/03/16

Answer all questions in a clear and concise manner. Remember that answers without explanation or that are poorly presented may not receive full credit.

**1.** Using l'Hospital's Rule, compute the following limit

$$\lim_{x\to 0} \frac{x^2}{1-\cos(x)}$$

**(a)** 0

**(b)** 1

**(c)** 2, using l'Hospital's Rule twice

**(d)** $\frac{1}{2}$

**(e)** L'Hospital's Rule doesn't apply

**2.** A shipping box has a square base of sidelength $x$ meters. The height of the container is $y$ meters. The post office will only accept the box if the sum of the height and the perimeter of the base is equal to 12 meters.

(a) Write down a function which gives the volume of the box in terms of $x$. Give the domain of the function.
In order to be shipped, the box must satisfy $y + 4x = 12$, or $y = 12 - 4x$. The volume of of the box is given by

$$V(x) = x^2 y = x^2(12 - 4x) = 12x^2 - 4x^3.$$

Since $x$ and $y$ are edge lengths, then $x \geq 0$ and $y = 12 - 4x \geq 0$, which implies $3 \geq x$. Thus the domain of $V$ is $[0, 3]$.

(b) Find the dimension $x$ and $y$ of the box with the largest possible volume.
To maximize $V$, we set $V'(x) = 0$. Thus

$$V'(x) = 24x - 12x^2 = 12x(2 - x) = 0$$

yields solutions $x = 0, 2$. Since $V(0) = 0$, the dimensions that yield the largest possible volume are $x = 2$ and $y = 4$.

(c) Explain why this yields the largest possible volume.
Since $V(0) = 0$ and $V(3) = 0$, the Closed Interval Method gives the maximum volume is obtained at $x = 2$.