MA 113 Calculus I, Fall 2016

Written Assignment #2

Due Friday, September 9, 2016, at beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

S_n is a regular n-gon inscribed in a circle of radius 1. (A regular n-gon is a polygon with n sides and where each of the sides has the same length and the angles around the n-gon are all equal. For example, a square is a regular 4-gon, and an equilateral triangle is a regular 3-gon.) The goal of this problem is to compute the perimeter p_n of the regular n-gon S_n and then try to compute $\lim_{n\to\infty} p_n$.

1. (1 point) Carefully draw a diagram showing a regular 6-gon (or hexagon) inscribed in a circle of radius 1. Use geometric reasoning to compute the perimeter p_6 of this hexagon.

2. (2 points) Draw a radius from the center of the circle to each vertex of S_n. This divides S_n into n congruent isosceles triangles. What is the measure of the angle θ (in radians) at the center of the circle for each of these triangles?

3. (3 points) Carefully explain why the length of the base of each of these isosceles triangles is equal to $2\sin(\frac{\pi}{n})$. (From the center of the circle, draw a perpendicular line to the base of the isosceles triangle, and then express the length of the base in terms of an appropriate trigonometric function and angle.)

4. (2 points) Find a formula for the perimeter p_n using the answer in (3). Verify that for $n = 6$, your formula agrees with your answer in (1).

5. (2 points) Try to find $\lim_{n\to\infty} p_n$. You could use your calculator to evaluate p_n for larger and larger values of n. Can you guess the correct limit by using geometric reasoning and by considering how the picture changes as n becomes arbitrarily large? Is your guess consistent with the numerical data given by your calculator?